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Why Do Firms Smooth Earnings?*

We explain why a firm may smooth reported earnings. Greater earnings volatility leads to a bigger informational advantage for informed investors over uninformed investors. If sufficiently many current shareholders are uninformed and may need to trade in the future for liquidity reasons, an increase in the volatility of reported earnings will magnify these shareholders' trading losses. They will, therefore, want the manager to smooth reported earnings as much as possible. Empirical implications are drawn out that link earnings smoothing to managerial compensation contracts, uncertainty about the volatility of earnings, and ownership structure.

I. Introduction

Corporate earnings management has been much in the news lately. For example, *Business Week* has recently run two cover stories, one titled "Who Can You Trust?" (October 5, 1998) and the other titled "The Numbers Game" (May 14, 2001), that suggest that the credibility of earnings reports is being eroded by earnings management. Arthur Levitt, Jr., chairman of the Securities and Exchange Commission (SEC), commented in 1998: "Too many corporate managers, auditors, and analysts are participants in a game of nods and winks. In the zeal to satisfy consensus earnings estimates and project a smooth earnings path, wishful thinking may be winning the day over faithful representation."[1]

Earnings management means manipulating reported earnings so that they do not accurately represent economic earnings at every point in time. Earnings smoothing is a special case of earnings management involving intertemporal smoothing of reported earnings relative to economic earnings; it attempts to make earnings look less variable over time. Earnings smoothing is extensively documented (see Beidlerman 1973; Bannister

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1. See *CPA Journal* (December 1998), pp. 14–19, quote on p. 14. See also Collingwood (2001) for a discussion of the evidence on earnings management.

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and Newman 1996; Subramanyam 1996). Moses (1987) studies how various firm-specific factors affect the extent of earnings smoothing. This raises the question we address: why is earnings smoothing so prevalent?

If earnings are being smoothed, reported earnings must be sometimes higher than economic earnings and sometimes lower. It is not difficult to see why managers may want to report inflated earnings. But it is a lot harder to explain why a manager reports lower earnings than what he observes. Yet, numerous such instances have recently been discussed. For example, in 1998, the SEC delayed approval of the acquisition of Crestar Financial Corporation by SunTrust Banks, Incorporated until the company agreed to reduce loan loss reserves by $100 million and restate higher earnings for the past 3 years.[2] The SEC also criticized W. R. Grace and Company for underreporting its 1998 profits by $20 million. The SEC alleged that the company was attempting to exploit apparently diminishing marginal returns to reported earnings. When reported earnings are high, reporting even higher earnings tends to elicit a relatively small positive market reaction. The company may therefore want to "hide" some of its current earnings for reporting in a future period when earnings are lower and the marginal impact of a higher report is greater.[3]

Earnings smoothing can be either "artificial" or "real." Real smoothing involves decisions that affect cash flows and dissipate firm value. Examples include changing the timing of investments and providing promotional discounts or vendor financing to risky customers to pump up sales toward the end of the quarter. By contrast, artificial smoothing does not affect cash flows. This kind of smoothing is achieved primarily by using the reporting flexibility provided by Generally Accepted Accounting Principles (GAAP).[4] Real smoothing has costs that are obvious, whereas artificial smoothing has costs that are subtler, such as those related to loss of credibility or consumption of the manager's time in such activities.

What the manager smooths in our model is the random variable that is the basis for setting the firm's price. Thus, it can be interpreted quite broadly as anything that goes into a standard valuation model, such as "dividends," "cash flow," or "economic earnings." While we choose to think of it as earnings, the main question is whether the manager has the discretion to disguise its true value within the boundaries of GAAP. The answer is yes. It is true that the cash flow number reported under GAAP may limit the manager's discretion over what earnings to report. But the cash flow reported under GAAP is not the construct investors use to value a stock, a contention strongly supported by the voluminous literature in accounting that claims that cash flow is not

2. See *American Banker* (March 11, 1999), p. 1.

3. See *Boston Globe* (December 23, 1998), p. D1.

4. Bannister and Newman (1996) and Defond and Park (1997) use discretionary accruals as estimators of smoothing. Moses (1987) considers accounting changes as a smoothing device. Michelson, Jordan-Wagner, and Wooton (1995) consider the ratio of coefficient of variation in income variable to the coefficient of variation in sales as a measure of smoothing.

a sufficient statistic for earnings for valuation purposes (see, e.g., Dechow [1994] and the references therein).

We now explain artificial smoothing. The basic intuition is as follows: in a perfect world with symmetric information, the volatility of the firm's earnings will be irrelevant if the shareholders are risk neutral or if they are risk averse but can costlessly diversify away their exposure to the firm. So we start with the assumption that there is some valuation-relevant information about the firm that cannot be costlessly and credibly communicated to shareholders. Investors can acquire this information at a personal cost, or they can choose to remain uninformed.

Some investors may have to sell their shares in the future due to (exogenous) liquidity reasons, such as an unexpected contingency. We assume that these liquidity investors are mostly shareholders of the firm. As they trade with informed investors, they lose money on average. In fact, it is their trading loss that compensates the informed investors for their information acquisition cost. Competition among the informed investors causes their expected trading profit to equal their information acquisition cost, producing zero ex ante expected profits. Thus, the resources devoted to information acquisition are a welfare loss absorbed mostly by the firm's shareholders.

The key to the analysis is that, when the volatility of the firm's earnings is high, private information about the firm is more valuable, and more investors become informed.[5] This means higher expected losses for shareholders who trade for liquidity. Shareholders, therefore, abhor earnings volatility and pay less for firms with higher earnings volatility.

The manager responds by smoothing earnings to affect market perceptions of earnings volatility and hence the firm's stock price. However, the market understands this in equilibrium and is not fooled. This means that there is no overall benefit from smoothing in equilibrium. The phenomenon persists nonetheless because not smoothing when the market expects smoothing can result in the firm's stock price being lower than its true value. It is interesting that what causes smoothing in our analysis is the manager's concern about long-term stock price performance rather than just the current stock price. A "myopic" manager would simply inflate earnings.

Smoothing reduces the expected value of the time-series volatility of reported earnings. The effect of smoothing on volatility is state contingent in that it depends on the shocks to earnings realized in the future. We assume that these shocks are serially uncorrelated, so smoothing reduces measured time-series volatility of earnings because shocks in successive periods tend to offset each other.[6] If the shocks are positively serially correlated, it is possible for smoothing in early periods to increase the variability of reported earnings in later periods.

5. Hereafter, earnings will mean economic earnings and not reported earnings.

6. As an example, let us consider a firm with stationary earnings with a mean of 100 and an unknown volatility. Suppose the manager wants to smooth earnings. If the earnings realization is x_1 in the first period, what can the manager do to make earnings look as smooth as possible? If the firm lives for two periods, an econometrician will estimate volatility by the standard

The existing literature has provided alternative explanations for earnings smoothing. Barnea, Ronen, and Sadan (1975) argue that earnings smoothing is a signaling device. In an overlapping generations model, Dye (1988) shows that current shareholders may demand earnings smoothing to influence perceptions of potential shareholders about firm value when the manager's contract with current shareholders is unobservable. Fudenberg and Tirole (1995) assume that management derives incumbency rents from continuing in the firm. Management can minimize the probability of being fired by developing a smooth performance record because the decision to fire or retain depends more on current performance than on past performance. Dividend and earnings smoothing arise since these are criteria for performance judgment.

Trueman and Titman (1988) point out that high perceived earnings volatility increases the perceived bankruptcy probability of the firm and hence its borrowing cost, so earnings smoothing is cost minimizing. Lambert (1984) explains real smoothing in a two-period moral hazard setting where the optimal contract has second-period managerial compensation increasing in first-period output. Thus, when first-period performance is good, the marginal utility of consumption in the second period is low for the manager, and he reduces effort. Rozycki (1997) uses the convexity of the tax code to explain smoothing.

What distinguishes our work from the existing literature is that earnings smoothing is not driven by issues related to managerial self-interest, tax incentives, or leverage concerns. Rather, it is solely the consequence of the manager trying to increase his firm's stock price by reducing the potential loss shareholders may suffer when they trade for liquidity reasons.[7] This is not to say that the factors examined by others are unimportant. Taken together with the

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deviation of the reported earnings. If y_1 and y_2 are the reported earnings in the two periods, their estimate of volatility will be $(y_1 - 100)^2 + (y_2 - 100)^2$. If the manager reports y_1 instead of x_1 in the first period, he must report $y_2 = x_1 + x_2 - y_1$ in the second period. The estimate of volatility will be $(y_1 - 100)^2 + (x_1 + x_2 - y_1 - 100)^2$. Its expected value, $(y_1 - 100)^2 + (x_1 - y_1)^2 + \mathrm{Var}(x_2)$, is minimized when the manager chooses $y_1 = (x_1 + 100)/2$. Thus, the manager divides total expected earnings equally across two periods. For instance, if the realized earnings are 110 in the first period, the manager "spreads" the positive shock of 10 over two periods and reports 105 in the first period. This minimizes the manager's expectation of the volatility estimate that an econometrician will calculate. Smoothing, however, does not always reduce the volatility estimate calculated by the investors. If the earnings are realized to be 110 in first period, the manager optimally reports earnings of 105. If second-period earnings are x_2, the manager will report $x_2 + 5$. The volatility estimate with smoothing is $(105 - 100)^2 + (x_2 + 5 - 100)^2$. The estimate without smoothing would be $(110 - 100)^2 + (x_2 - 100)^2$. If the second-period earnings turn out to be 105 or lower, smoothing reduces the volatility estimate. However, if the second-period earnings turn out to be higher than 105, smoothing increases the volatility estimate. Since realized earnings are more likely to be below 105 than above (the mean is 100), smoothing is more likely to reduce the volatility estimate than to raise it. Further, the reduction in the volatility estimate is likely to be larger than the increase in the volatility estimate. Thus, the manager can minimize the expected value of the estimate of volatility by smoothing.

7. The intuition is reminiscent of the idea in Brennan and Thakor (1990) that, in choosing between a stock repurchase and a dividend as a cash disbursement mechanism, the manager takes into account the fact that a repurchase forces uninformed shareholders to possibly trade against informed shareholders, whereas a dividend does not cause one group of shareholders to be disadvantaged relative to another.

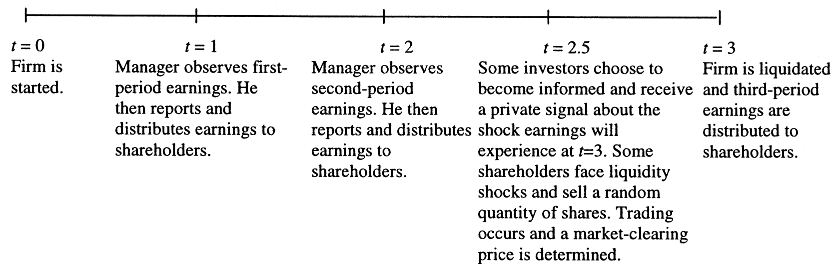

FIG. 1.—Time line of events

existing literature, our analysis suggests that there may be many reasons why firms smooth earnings.

Section II lays out the model. Section III discusses the information acquisition and liquidity trading process and relates it to perceptions about firm's earnings stream. Section IV analyzes earnings smoothing, and Section V provides a numerical example. Section VI examines robustness issues, and Section VII concludes. All proofs are in the appendix.

II. Model

In this section, we describe the model in detail, including the sequence of events, the agents involved, and who knows what and when. See figure 1 for a summary.

A. *Time Line*

There are four dates in the model: $t = 0$, $t = 1$, $t = 2$, and $t = 3$, corresponding to three time periods: the first ($t = 0$ to $t = 1$), the second ($t = 1$ to $t = 2$), and the third ($t = 2$ to $t = 3$). A firm was started in the past (say, at $t = 0$). The earnings of the firm for the three periods of its existence are realized at $t = 1$, 2, and 3, respectively. After the earnings realization at $t = 3$, the firm is liquidated. Sometime during the third period (say, at $t = 2.5$), there is liquidity trading in the firm's shares. We will explain this later in this article.

B. *The Players*

The firm is all-equity financed, with the number of shares outstanding normalized to one. The manager of the firm privately observes the economic earnings e_{at} in period t and then reports e_{rt} to others. Besides the manager and the shareholders, there are many competing investors in the market who can acquire costly private information about the firm. Since we use a market microstructure model of trading similar to Boot and Thakor's (1993), we adopt their assumption that all investors are capital constrained and cannot short-sell or trade on margin. Any investor choosing not to get informed can still trade competitively based on publicly available information. Thus, prior to the information acquisition, the players are the manager and the investors, some of whom are shareholders.

After information acquisition, we have the manager and the investors, who fall into two groups: informed and uninformed. Current shareholders are assumed to be uninformed, and there are other uninformed investors as well who do not currently own the firm's stock. For simplicity, in the post–information acquisition stage, we shall henceforth label the "informed investors" as *speculators* and the "uninformed investors" who are not shareholders as the *market maker*. Since separation among different groups of investors occurs after information acquisition, we shall refer to all agents other than the manager in the pre–information acquisition stage as *investors*. The market maker in the post–information acquisition stage should be thought of as a "representative" uninformed investor. The market maker sets a competitive price at $t = 2.5$ using the common information set of all uninformed investors and clears the market. The manager maximizes the shareholders' wealth. We assume universal risk neutrality.

C. Earnings Distribution

The earnings stream of the firm is stationary. The form of the earnings distribution is common knowledge, but the parameters of this distribution are unknown. The earnings distribution at time t is

$$e_{at} = \begin{cases} \mu + \delta & \text{if } \lambda_t = \text{ high (positive shock)} \\ \mu - \delta & \text{if } \lambda_t = \text{ low (negative shock),} \end{cases} \tag{1}$$

$$\lambda_t | \lambda_1, \ldots, \lambda_{t-1} = \begin{cases} \text{high with probability } 0.5 \\ \text{low with probability } 0.5, \end{cases} \tag{2}$$

for $t = 1, 2,$ and 3, and so the distribution is time invariant.

The parameters μ and δ are the mean and the volatility of the distribution, and they are unknown to everyone, including the manager. The common prior beliefs about the distribution of μ and δ are

$$\begin{aligned} \delta &\sim g(\delta) \quad \text{with support} \quad (\underline{\delta}, \overline{\delta}), \\ \mu &\sim h(\mu) \quad \text{with support} \quad (\underline{\mu}, \overline{\mu}), \end{aligned} \tag{3}$$

and μ and δ are independently distributed.[8]

8. Note that, while the expected earnings parameter, μ, in our model is constant, in reality it may be time varying.

D. Managerial Discretion

The manager reports only a single number, the firm's earnings. He has some discretion in choosing what to report at $t = 1$. The manager's report can differ from the economic earnings by, at most, d, where $d > 0$ is common knowledge. Thus, for simplicity, we are assuming that earnings can be costlessly manipulated by an amount up to d but that it is prohibitively costly to manipulate more. This captures, in a stylized way, the notion that the manager has limited reporting discretion.

In exercising his reporting discretion, the manager "rolls over" an earnings "surplus" or "deficit" to another time period. We assume that the manager cannot "roll over" earnings forever and must settle accounts at $t = 2$.[9] Thus, misreporting at $t = 1$ must be followed by an offsetting misreporting at $t = 2$.

The discount rate is zero. Therefore, any inflation of earnings by $1 at $t = 1$ must cause a deflation of earnings by $1 at $t = 2$, and vice versa. There is no investment or share repurchase, and reported earnings are distributed as dividends to the shareholders. Since total earnings equal total cash flows over the life of the firm, total dividends paid equal total cash flows. We assume that the firm is not cash constrained, and so it can pay dividends in the first period that exceed first-period cash flow. Everyone sees the earnings for the third period at $t = 3$, when the firm is liquidated and the proceeds are distributed to the shareholders.

The assumption that the manager reports earnings as the only measure of firm performance is a simplification. In practice, investors receive multiple reports, including reported earnings and cash flows. These reports provide cross-checks that limit the manager's ability to reduce the information content of any report. However, all that we need is that an earnings report contain information that is valuable to investors but cannot be recovered from cash flows.[10]

As mentioned in the introduction, when firm performance is reported at discrete points in time, current earnings may contain information to predict future cash flows that current cash flows do not contain. There is evidence that earnings and cash flow both have incremental information over each other

9. We do not consider a rollover of earnings to $t = 3$ in order to simplify the analysis. This assumption narrows the set of reporting strategies available to the manager. We believe that this assumption is not critical for our results.

10. The extent to which earnings can be artificially smoothed is limited by realized cash flows, which cannot be misreported; there are only so many earnings values compatible with realized cash flows. But earnings cannot be inferred unambiguously from cash flows, and so the manager has some discretion in reporting earnings. Although economic earnings and cash flows converge in the long run, they differ in the short run due to revenue recognition discretion, accruals, extraordinary items, etc.; unlike cash flows, earnings attempt to match costs and benefits through time. Thus, earnings carry information that cannot be inferred directly from cash flows. To see this, consider a firm that makes a sale of $1 million and delivers goods on December 24, 2000. The sale revenue will be received in the year 2001. The firm incurs period costs (in cash) of $900,000. The firm uses calendar years as financial years. If we ignore taxes, these transactions reduce year 2000 cash flow by $900,000 and increase earnings by $100,000 if the sale is recognized in the year 2000 report.

in predicting future cash flows (see Bowen, Burgstahler, and Daley 1987; Dechow 1994).[11] Thus, introduction of other reports such as cash flow would not qualitatively change the results. The earnings report in this model should be interpreted as the information provided by the manager that is not already contained in cash flow.

E. Liquidity Selling

During period 3, some shareholders face liquidity shocks and are forced to sell their shares; these shares are sold at the market-clearing price at $t = 2.5$, and the supply (shares sold) is stochastic. The supply l in dollar terms has the distribution

$$l \sim f(l) \text{ with support } (\underline{l}, \bar{l}), \tag{4}$$

and it is independent of the earnings history (economic or reported) of the firm.

F. Information Structure

We first specify the information sets of the players in the pre–information acquisition stage. Let ϕ^j denote the information set of the manager and $\hat{\phi}^j$ the information set of investors just after $t = j$, where $j = 1$ or 2. Everyone remembers past information, so the information sets do not decay over time. Everyone starts with common prior knowledge about the distributions of μ and δ. The manager gets to see the economic earnings e_{aj} at $t = j$, where $j = 1$, 2, and 3. Investors do not observe e_{a1} and e_{a2}. They see only the earnings reports e_{r1} and e_{r2} at $t = 1$ and $t = 2$, respectively. The economic earnings at $t = 3$, e_{a3}, are seen by everyone during liquidation. Investors set the price P_j of the shares of the firm at $t = j$, where $j = 1$ or 2, using the information set $\hat{\phi}^j$, and this price is observed by everyone. The price P_j depends deterministically on $\hat{\phi}^j$, so it is also a part of $\hat{\phi}^j$. We will use E^j as the expectation conditional on the information set ϕ^j of the manager and $\hat{E}^j$ as the expectation conditional on the information set $\hat{\phi}^j$ of investors.

Now we consider the evolution of information sets in the post–information acquisition stage. Each investor in this economy starts with \$(1 + M). Each can incur a fixed cost of \$$M$ to obtain a signal γ about the firm's prospects at $t = 2.5$ before liquidity trading. The label *speculators* is used for those who choose to acquire γ. The signal γ tells with certainty whether the firm will experience a positive or a negative shock to earnings at $t = 3$. Note that the signal does not convey any information about the value of μ or δ or the

11. An investor who observes only the firm's cash flow sees a picture different from that seen by an investor who sees only the earnings. Earnings and cash flow together tell us more than does either alone. In particular, if future sales are correlated with current sales, earnings may provide information about future cash flows that the current cash flow does not. This illustrates that earnings may have valuation-relevant information not contained in cash flow. It is on this information that the manager in our model exercises his reporting discretion. Thus, the manager artificially smooths earnings even when he cannot smooth cash flows.

exact level of earnings. The information set of speculators is $\hat{\phi}^2 \cup \{\gamma\}$ before trading occurs. Speculators cannot observe the liquidity supply while making their trading decisions. The market maker observes demand or supply that is net of liquidity shareholders' supply and any demand from speculators. Let S denote the net supply seen by the market maker ($S < 0$ if there is net demand), who uses information $\hat{\phi}^2 \cup \{S\}$ to set a market-clearing price.[12] We denote by θ the measure of speculators; θ is common knowledge at the time of liquidity trading because everyone can solve the optimization problem of an investor who has to choose whether to become informed or not, and thus determine how many investors have become (informed) speculators.

Investors must decide whether to become informed or not before liquidity shocks occur. We assume for simplicity that no shareholder becomes informed.[13] The implications of relaxing this assumption are discussed in Section VI.

We want to analyze the behavior of the investors and the manager. The first step is to study what happens in liquidity trading at $t = 2.5$ and how shareholders are affected. The effect of liquidity trading on shareholders depends on the mean and the volatility of the firm's earnings. At $t = 1$ and $t = 2$, investors use their perceptions of the firm's mean and volatility to anticipate the outcome of the liquidity trading at $t = 2.5$ and set share prices. The manager knows this, and his objective is to maximize the prices of the shares. He does this by using his earnings-reporting discretion to influence investors' perceptions.

III. Equilibrium Analysis of Liquidity Selling

The manager reports earnings at $t = 1$ and $t = 2$. Investors try to infer the mean μ and the volatility δ of the firm's earnings from these reports. Let the expectations of the mean and the volatility based on investor perceptions be $\hat{\mu}$ and $\hat{\delta}$, respectively. The next section discusses how these perceptions are formed.

We show below that there is a Nash equilibrium in which the measure of investors who become (informed) speculators depends on the characteristics of the firm's earnings. The reason is that these characteristics determine the ex post trading profits of the speculators at $t = 2.5$ and hence the marginal return to getting informed. The trading profits of speculators come at the expense of shareholders who sell for liquidity reasons and systematically suffer

12. The market maker represents uninformed investors who trade competitively to clear the market. However, we do not rule out current shareholders from trading and competing with other uninformed investors. In fact, the shareholders will be required to clear the market when the demand by informed investors exceeds the supply by liquidity-seeking shareholders.

13. This assumption is not critical to the analysis, as we discuss later. We can justify it, however, by assuming that current shareholders have exorbitant information acquisition costs or are otherwise wealth constrained.

losses in equilibrium. Thus, shareholders' expected losses depend on the characteristics of earnings.

The net supply of shares, S, observed by the competitive market maker depends on the liquidity shareholders' supply, l, and any demand by speculators. We have

$$S = S(l, \gamma, \theta) = \begin{cases} l & \text{if } \gamma \text{ signals negative shock} \\ l - \theta & \text{if } \gamma \text{ signals positive shock.} \end{cases} \tag{5}$$

As we explain later, speculators demand shares only when the private signal is favorable. If the market maker knew the liquidity supply l or the signal γ, he could figure out whether earnings would experience a positive or a negative shock at $t = 3$ and set price equal to the expected value of earnings, $\hat{\mu} + \hat{\delta}$ or $\hat{\mu} - \hat{\delta}$. However, the market maker observes only the net supply S and not l or γ. Therefore, he uses prior beliefs about the distribution of liquidity supply to arrive at an unbiased Bayesian posterior estimate of share value. The price function is

$$P(S, \theta) = \frac{\frac{1}{2} f(S + \theta)(\hat{\mu} + \hat{\delta}) + \frac{1}{2} f(S)(\hat{\mu} - \hat{\delta})}{\frac{1}{2} f(S + \theta) + \frac{1}{2} f(S)}. \tag{6}$$

The net supply S equals the liquidity supply l if speculators observe a low signal and submit no demand. The likelihood of l being S and the signal being low is $f(S)/2$, and the firm value in this case is $\hat{\mu} - \hat{\delta}$. If speculators observe a high signal, they demand θ, and so the liquidity supply must be $S + \theta$. The likelihood of this is $f(S + \theta)/2$, and the firm value in this case is $\hat{\mu} + \hat{\delta}$. Using Bayes's rule and setting price equal to the firm's expected value yields (6).

ASSUMPTION 1. The density function of liquidity supply, f, is log-concave.[14]

Assumption 1 provides a sufficient condition for an economically meaningful pricing function. A log-concave density function of liquidity supply ensures that price is a decreasing function of net supply of shares.[15]

From (6), we see that the share price is always between $\hat{\mu} - \hat{\delta}$ and $\hat{\mu} + \hat{\delta}$. Therefore, a speculator will never buy when he sees an unfavorable signal and will always buy when he sees a favorable signal. As the two cases are equally likely, the speculator's expected trading profit is

$$m(\hat{\mu}, \hat{\delta}, \theta) = \frac{1}{2} \int_{\underline{l}}^{\bar{l}} \frac{(\hat{\mu} + \hat{\delta}) - P(l - \theta, \theta)}{P(l - \theta, \theta)} f(l) dl. \tag{7}$$

14. The density function f is log-concave if $\log[f(x)]$ is concave in x. Some of the standard distributions satisfying this property are uniform, (truncated) normal, and (truncated) exponential.

15. See lemma A1 in the appendix for the proof.

Each speculator submits a \$1 demand for shares when he observes a high signal. If the shareholders' liquidity supply is l, the net supply S observed by the market maker is $S = l - \theta$, and the price is set at $P(l - \theta, \theta)$. Each speculator ends up buying $1/P(l - \theta, \theta)$ shares and makes a profit of $(\hat{\mu} + \hat{\delta}) - P(l - \theta, \theta)$ per share. This trading profit is conditional on liquidity supply l and a high signal. The expected trading profit is obtained by taking an expectation over l and then multiplying by 1/2, the probability of a high signal.

The measure of speculators, θ, taken as exogenous thus far, is uniquely determined in equilibrium by a condition of competition among investors that says that the marginal speculator must earn zero ex ante expected profit, net of his information acquisition cost. Since we assume the same information acquisition cost for all investors, all speculators make zero expected profit in equilibrium and investors are indifferent between becoming informed and staying uninformed. A key result is the following.

LEMMA 1. The expected trading profit of each speculator is decreasing in the measure of speculators.

The intuition is that, as more investors get informed, more of their information enters the price. This reduces mispricing as well as the trading profit of each speculator that is based on this mispricing. We can now define equilibrium.

DEFINITION 1. A Nash equilibrium is a measure θ^* of speculators such that $m(\hat{\mu}, \hat{\delta}, \theta^*) = M$; a net supply of shares $S(l, \gamma, \theta^*)$, as in (5); and a market clearing price $P(S, \theta^*)$, as in (6).

We will now show that there is a unique equilibrium as defined above. To avoid the uninteresting case in which the cost of getting informed is too high for anyone to get informed, we make the following assumption.

ASSUMPTION 2. $M < \hat{\delta}/2\hat{\mu}$.

PROPOSITION 1. There is a unique Nash equilibrium with the measure of speculators, $\theta^* = \theta^*(\hat{\mu}, \hat{\delta})$.

Investors know the distribution of liquidity supply, which enables them to compute the expected trading profit of a speculator as a function of the measure of speculators θ, using (7). The expected trading profit of a speculator is decreasing in θ, and so the number of speculators adjusts to a level such that the expected trading profit for the marginal speculator equals M. Those who become speculators in equilibrium receive a signal about the earnings shock the firm will receive and demand shares only if the signal is favorable. The market maker sees aggregate demand and sets a market-clearing price such that he expects zero profit.

Liquidity-seeking shareholders suffer (ex post) losses on average when they trade. The reason is that their trades are induced by exogenous liquidity shocks, whereas those they trade with are trading strategically based on information. If there are no speculators, the price does not depend on the order flow because there is nothing to be learned from the order flow. Good and bad firms are priced alike. Bad firms are overpriced and good firms are underpriced, but, since all investors are trading on a "level playing field," nobody makes expected losses or gains. The presence of speculators reduces mispricing because their trades cause their information to be noisily reflected in prices. The mispricing does not vanish altogether, however, because the number of speculators is finite and the market maker cannot distinguish between liquidity trades and informationally motivated trades. In fact, this mispricing is needed to attract some to become speculators since their information acquisition costs are recovered from the expected trading profits generated by mispricing. If there were no cost of information, the number of speculators would be so large that order-flow would reveal all private information and there would be no mispricing. The order flow would also be perfectly revealing if there were no shareholders seeking liquidity. We now characterize the dependence of the measure of speculators on the characteristics of the firm's earnings stream.

PROPOSITION 2. (*a*) The expected trading profit of a speculator, m, is an increasing function of the ratio of the perceived earnings volatility $\hat{\delta}$ to the perceived earnings mean $\hat{\mu}$; (*b*) the measure of speculators, θ^*, depends on the ratio of the earnings volatility to the earnings mean, that is,

$$\theta^*(\hat{\mu}, \hat{\delta}) = \theta^*(\hat{\delta}/\hat{\mu}). \tag{8}$$

Further, θ^* is an increasing function of this ratio, $\hat{\delta}/\hat{\mu}$.

The intuition behind proposition 2 is as follows: the higher the volatility of the earnings, the higher is the informational advantage of the speculators over other investors. The reason is that the signal the speculators privately observe reveals whether earnings will experience a positive or negative shock, and the size of this shock is increasing in the volatility. Thus, if the measure of speculators is fixed, their expected trading profit increases with earnings volatility. Competition among speculators, however, results in more investors becoming speculators when perceived earnings volatility is high, so that the marginal speculator's expected trading profit equals the cost of acquiring the signal.

An increase in the earnings mean has a different effect, however. Since the signal the speculators receive contains no information about mean earnings, they have no informational advantage over others when it comes to the mean. But as the mean μ increases, keeping the size of the earnings shock δ fixed, the impact of δ relative to μ on the firm's stock price decreases. This diminishes the informational advantage of the speculators and hence their ex post trading profits. This causes the measure of speculators to shrink as μ, and, hence, the volatility-to-mean ratio increases.

The following result shows that the equilibrium measure of speculators determines the expected trading losses of liquidity-seeking shareholders at $t = 2.5$.

LEMMA 2. The expected trading losses of shareholders equal the information acquisition cost of speculators.

The expected trading profit of any speculator equals the marginal speculator's cost of acquiring the signal in equilibrium. All investors face the same information acquisition cost, so the expected trading profits of all speculators equal their information acquisition cost. The profits come entirely at the expense of liquidity-seeking shareholders because a market maker makes zero expected profit when he clears market. The expected losses of shareholders thus equal the information acquisition cost of speculators. This means that these expected losses are proportional to the measure of speculators. In light of proposition 2, the expected losses of liquidity-seeking shareholders are thus increasing in the earnings volatility-to-mean ratio.

IV. Earnings Smoothing

To understand the manager's incentives to smooth earnings, we need to discuss the form of his compensation contract, the choices available to him in reporting earnings, and the impact of earnings reports on investors' perceptions of the firm's earnings stream. The manager will choose a reporting strategy that maximizes his expected compensation, taking into account the effect of earnings reports on the investors' perceptions and, consequently, on his compensation.

A. Manager's Compensation

Since moral hazard due to unobservable effort is precluded by assumption, current shareholders design the compensation contract to induce the manager to report earnings to maximize the value of their shares. The compensation contract may, in principle, be a function of all the observable variables. The shareholders may make the compensation contingent on the reported earnings in the three periods and on the prices after earnings reports at $t = 1$ and $t = 2$.

The shareholders' claims are residual to the manager's compensation. To avoid the complication of considering the effect of the manager's compensation on the firm's cash flows, we assume that the manager's compensation is very small as compared with the earnings level. This allows us to ignore the effect of a change in the manager's compensation on the earnings.

We assume that the manager's compensation is proportional to

$$\alpha(e_{r1} + P_1) + (1 - \alpha)P_2, \tag{9}$$

where $0 \leq \alpha \leq 1$. The compensation is linked to prices at $t = 1$ and $t = 2$. The earnings and price at $t = 1$ are added to remove the obvious distortions in the manager's reporting that would arise if a dollar paid as dividend were weighted differently from a dollar saved for the future.

B. Managerial Discretion

The earnings reported by the manager may vary from the economic earnings by an amount $d > 0$; that is, $| e_{a1} - e_{r1} | \leq d$. The manager, however, has to "clear the position" created by this manipulation in earnings. If reported earnings at $t = 1$ were higher than economic earnings by an amount x, then reported earnings at $t = 2$ must be less than the economic earnings by an amount x. Similarly, lower-than-economic first-period reported earnings must be followed by higher-than-economic second-period earnings. The total economic earnings in the first two periods thus equal the total reported earnings in the first two periods:

$$e_{a1} + e_{a2} = e_{r1} + e_{r2}. \tag{10}$$

The discretion available to the manager is assumed low enough to permit investors to infer from the reported earnings whether the earnings shock was positive or negative in any period.

ASSUMPTION 3.

a) $(\bar{\mu} - \underline{\mu}) < 2(\underline{\delta} - d)$;
b) $(\bar{\mu} - \underline{\mu}) < \underline{\delta}$.

Condition *a* ensures that investors can always use reported earnings at $t = 1$ to infer if there is a positive or a negative shock. This is because the highest earnings report possible with a negative shock is less than the lowest earnings report possible with a positive shock; that is,

$$\bar{\mu} - \underline{\delta} + d < \underline{\mu} + \underline{\delta} - d.$$

Thus, investors use the following rule to infer the kind of shock experienced by earnings at $t = 1$:

$$\lambda_1 = \begin{cases} \text{high} & \text{if } e_{r1} \geq \underline{\mu} + \underline{\delta} - d \\ \text{low} & \text{if } e_{r1} \leq \underline{\mu} + \underline{\delta} + d. \end{cases} \tag{11}$$

The manager, however, observes the economic earnings and thus updates as follows:

$$\lambda_t = \begin{cases} \text{high} & \text{if } e_{at} \geq \underline{\mu} + \underline{\delta} \\ \text{low} & \text{if } e_{at} \leq \bar{\mu} - \underline{\delta}. \end{cases} \tag{12}$$

Condition *b* ensures that investors, after observing the reports for the first two periods, can infer whether the earnings at $t = 2$ experienced a positive or a negative shock. Investors use the following rule to determine the kind of earnings shock experienced at $t = 2$:

$$\begin{aligned} &\lambda_1 = \lambda_2 = \text{high if } \frac{e_{r1} + e_{r2}}{2} \geq \underline{\mu} + \underline{\delta} \\ &\lambda_1 = \lambda_2 = \text{low if } \frac{e_{r1} + e_{r2}}{2} \leq \bar{\mu} - \underline{\delta} \\ &\lambda_1 \text{ and } \lambda_2 \text{ are different if } \underline{\mu} \leq \frac{e_{r1} + e_{r2}}{2} \leq \bar{\mu}. \end{aligned} \tag{13}$$

C. Perceptions about the Earnings Stream

Everyone has common prior beliefs about the parameters of the earnings stream, given by (1) and (3). The economic earnings value conveys some information about the mean μ and the volatility δ. When earnings experience a positive shock, the earnings value equals $\mu + \delta$. Therefore, a high economic earnings value is likely to be associated with a high mean and a high volatility. When earnings experience a negative shock, the earnings value equals $\mu - \delta$. Therefore, a high economic earnings value is likely to be associated with a high mean and a low volatility. The exact effect depends on the distributions of μ and δ. We assume that the uncertainty about the mean is low as compared with the uncertainty about the volatility in a sense made precise below.

Assumption 4.

a) $E[\delta \mid \mu + \delta = S]/E[\mu \mid \mu + \delta = S]$ is increasing in S.
b) $E[\theta^*(\delta/\mu) \mid \mu + \delta = S]$ is increasing in S, while $E[\theta^*(\delta/\mu)|\mu - \delta = S]$ is decreasing in S.
c) $E[\delta|\delta \geq \underline{\delta} - r, \mu + \delta = \bar{\mu} + \bar{\delta} - s] \leq \bar{\delta} - kr\ \forall s \geq r, r \leq 2d$, where $k \in (0, 1)$.
d) $(\mathrm{d}/\mathrm{d}S)E[\delta|\mu - \delta = S] \leq 0$, $(\mathrm{d}/\mathrm{d}S)E[\mu|\mu - \delta = S] \geq 0$.

Conditions in assumption 4 refer to rational beliefs formed after observing economic earnings. Condition *a* requires that, during positive earnings shocks, any increase in earnings causes a higher percentage increase in the expected volatility than in the expected mean of the earnings stream. Condition *b* requires that higher earnings during a positive shock or lower earnings during a negative shock indicate high volatility and change posterior distribution of δ/μ in such a way that expected measure of speculators θ^* increases. Condition *c* requires that the density functions of μ and δ are smooth and the uncertainty in volatility is high, so that, conditional on the sum $\mu + \delta$ (the economic earnings in a high-shock period) being low, the expected value of δ is sufficiently low.[16] Condition *d* requires that, with a negative earnings shock, a higher value of economic earnings leads to a perception of lower volatility and higher mean. Conditions *a*, *b*, and *c* are satisfied if the uncertainty in mean is low as compared with the uncertainty in volatility, while condition *d* is satisfied for most regular probability density functions.

Assumption 4 ensures that the manager's incentive to smooth earnings is

16. Assumption 4*c* can be replaced by the simpler condition $E[\delta \mid \mu + \delta = \bar{\mu} + \bar{\delta} - r] \leq \bar{\delta} - kr$ if density function g of μ is log-concave (see n. 14). This simpler condition requires that there be sufficient uncertainty in volatility so that a low earnings realization during positive shock indicates sufficiently low expected volatility. This leads to the condition in assumption 4*c* as

$$\begin{aligned} E[\delta \mid \mu + \delta = \bar{\mu} + \bar{\delta} - s, \delta \geq \bar{\delta} - r] &\leq E[\delta \mid \mu + \delta = \bar{\mu} + \bar{\delta} - r, \delta \geq \bar{\delta} - r] \\ &= E[\delta \mid \mu + \delta = \bar{\mu} + \bar{\delta} - r] \leq \bar{\delta} - kr \end{aligned}$$

for $s \geq r$. The first inequality follows from the property that, for random variables x and y, $E[y \mid x + y = S]$ is increasing in S if the probability density function of x is log-concave.

strong as compared with his incentive to inflate earnings. Earnings smoothing reduces the perceived volatility of earnings, and earnings inflation increases the perceived mean of earnings. The former strategy is effective when there is significant uncertainty about the volatility of earnings, and the latter is effective when there is significant uncertainty about the mean of earnings. The conditions in assumption 4 assert that the uncertainty about the volatility is high relative to the uncertainty about the mean, and so the manager's incentive to smooth earnings dominates his incentive to inflate earnings. We later show that the optimal compensation scheme sets $\alpha = 0$ in (9) when there is no trading between periods. In this case, the manager has no incentive to inflate earnings, and we do not need assumption 4 for our results. Assumption 4 provides sufficient conditions to make our analysis robust to small changes in the compensation contract. A special case of our model is that in which there is no uncertainty about the mean of the earnings stream. Assumption 4 holds trivially then, and all our results are sustained. However, we keep the setup general for two reasons. First, it illustrates the trade-offs that a manager faces between smoothing earnings and simply overreporting earnings. Second, it enables us to generate an empirical prediction about which firms are likely to smooth earnings and which are likely to inflate earnings.

Assumption 4 formalizes how the value of economic earnings affects investors' beliefs about μ and δ. Recall that investors observe only reported earnings, not economic earnings. The effect, however, is qualitatively the same if the manager's report is a monotone function of economic earnings. Suppose that the manager's reporting strategy is to report higher earnings, e_{r1}, for higher values of economic earnings e_{a1}. Then the beliefs formed by investors based on observing the earnings report have the following characteristics:

1. With a positive shock to earnings, higher reported earnings e_{r1} lead to higher $\hat{E}^1[\delta]/\hat{E}^1[\mu]$.
2. With a negative shock to earnings, higher reported earnings e_{r1} lead to higher $\hat{E}^1[\mu]$ and lower $\hat{E}^1[\delta]$.

These characteristics determine how reported earnings affect prices P_1 and P_2. Suppose that investors have some beliefs about the manager's reporting strategy. If the manager does not follow this strategy and reports higher earnings at $t = 1$, then he fools investors into believing that the mean level of earnings is higher. At $t = 2$, the manager will have to offset this effect, and then investors will form correct beliefs about the mean μ. Thus, the higher perception about μ has a positive effect on P_1 but no effect on P_2.

The effect of the manager's report on investors' perceptions about earnings volatility is not so straightforward. When there is a negative shock to earnings, a high report lowers perceived volatility δ. When there is a positive shock to earnings, a high report raises perceived volatility. A high volatility is undesirable from the perspective of shareholders because then they expect to lose more at $t = 2.5$, as discussed in the previous section. These expected losses affect prices P_1 and P_2. Thus, if the manager increases reported earnings, its effect on perceived volatility increases expected prices during negative shocks but decreases expected prices during positive shocks.

The manager's report affects P_1 and P_2 through its combined effect on the perceived mean and volatility of earnings. During a negative shock, a high earnings report has an unambiguously favorable effect on prices. During a positive shock, a high earnings report has a favorable effect on P_1 because of a higher perceived μ, but it has an unfavorable effect on both P_1 and P_2 because of a higher perceived δ. Since the manager's compensation is an increasing function of P_1 and P_2, he has an incentive to inflate earnings if earnings experience a negative shock, but the incentives are less clear if earnings experience a positive shock. In proposition 3, we specify conditions under which the manager wants to report low earnings during a positive shock to earnings. We show that the manager smooths the earnings to the maximum extent possible. The earnings report at $t = 1$ as a function of the economic earnings under this smoothing strategy is characterized by

$$e_{r1} = \begin{cases} e_{a1} - d & \text{if } \lambda_1 = \text{high} \\ e_{a1} + d & \text{if } \lambda_1 = \text{low.} \end{cases} \tag{14}$$

Investors are assumed to set the prices competitively equal to the expected value of future cash flows from a share. Thus, the pricing function is

$$\begin{aligned} P_1 &= \hat{E}^1[e_{a2} + e_{a3} + (e_{a1} - e_{r1}) - M\theta^*(\hat{E}^2[\mu], \hat{E}^2[\delta])], \\ P_2 &= \hat{E}^2[e_{a3}] - M\theta^*(\hat{E}^2[\mu], \hat{E}^2[\delta]). \end{aligned} \tag{15}$$

The prices in (15) are set as expected values of future cash flows to the shareholders conditional on the information available to them; what someone is willing to pay for a share depends on his expected payoff from owning that share. Given the possibility of a liquidity shock, a shareholder cannot be sure of holding the share until the terminal date. This means that the shareholder's expected payoff differs from the firm's expected terminal cash flow; it also depends on the possible trading losses the shareholder could incur, given a liquidity shock prior to the terminal date. The price at $t = 1$ equals the expected value of the economic earnings in the second and third periods plus any balance from the first-period's earnings (positive if the manager reported lower than economic earnings and negative if the manager reported higher than economic earnings) minus shareholders' expected liquidity-trading loss. The price at $t = 2$ equals the expected value of third-period earnings minus the expected liquidity-trading loss.[17]

Proposition 3. For sufficiently low values of α, the weight placed in

17. In this sense, our model is different from other market microstructure models (e.g., Kyle 1985; Boot and Thakor 1993) in which liquidity traders expect to lose money from trading in an ex ante sense and are thus irrational in participating. Here investors who did not own shares initially but choose to become (uninformed) shareholders expect ex ante to break even since their expected trading losses are absorbed by the firm through the way its stock is priced. Of course, those who owned the firm at the outset suffer a loss in wealth because the firm stock is priced lower this way.

the manager's compensation contract on the cum-dividend share price at $t = 1$, the following constitutes a sequential equilibrium for the game in the first two periods:

Manager's strategy: The manager smooths and reports earnings according to (14) and (10).

Investors' strategy: Investors price competitively according to (15), using (1) and (2) to form expectations about past and future earnings, with their beliefs about μ, δ, and λ_t as specified below.

Manager's beliefs: The manager observes economic earnings (only e_{a1} at $t = 1$ while both e_{a1} and e_{a2} at $t = 2$), uses (12) to infer λ_t, and uses (1) and (3) to form beliefs about the distributions of μ and δ.

Investors' beliefs: At $t = 1$, investors use (11) to infer λ_1. Their beliefs about economic earnings are

$$e_{a1} = \begin{cases} \min(e_{r1} + d, \bar{\mu} + \overline{\underline{\delta}}) & \text{if } \lambda_1 = \text{high} \\ \max(e_{r1} - d, \underline{\mu} - \delta) & \text{if } \lambda_1 = \text{low,} \end{cases}$$

and their beliefs about μ and δ are based on (1) and (3). At $t = 2$, investors use (11) and (13) to infer λ_1 and λ_2. Their beliefs about μ, δ, and economic earnings are these: if $\lambda_1 = \lambda_2$, then $e_{a1} = e_{a2} = (e_{r1} + e_{r2})/2$ and use (1) and (3) to form beliefs about μ and δ. If $\lambda_1 \neq \lambda_2$, then $\mu = (e_{r1} + e_{r2})/2$,

$$\delta = \begin{cases} \min(e_{r1} + d - \mu, \overline{\underline{\delta}}) & \text{if } \lambda_1 = \text{high} \\ \min(\mu - e_{r1} + d, \delta) & \text{if } \lambda_1 = \text{low,} \end{cases}$$

and use (1) to infer e_{a1} and e_{a2}.

The above proposition shows that when the manager's compensation relies sufficiently heavily on the price P_2 at $t = 2$, the manager smooths earnings to the maximum extent possible. Recall that the manager always has an incentive to overreport earnings when the economic earnings experience a negative shock at $t = 1$. However, when earnings experience a positive shock, a high report has two opposing effects. On the one hand, it decreases P_1 and P_2 due to a high perceived volatility. On the other hand, it increases P_1 due to a high perceived mean. When the impact of volatility is more significant (see assumption 4*a*) and the manager's compensation is very sensitive to price P_2, the first effect dominates, and the manager underreports earnings. Investors, however, rationally anticipate the smoothing behavior of the manager in equilibrium and infer economic earnings from reported earnings to form correct beliefs about the mean and the volatility.

It may seem surprising that smoothing would occur in a model in which there is a one-to-one mapping from economic to reported earnings and investors are not fooled by the smoothing in equilibrium. The reason for this result is that, given investors' equilibrium expectation that smoothing is taking place, any firm that does not smooth is worse off. Thus, what we have is a form of prisoner's dilemma. Smoothing is initially motivated by the firm's desire to reduce perceived earnings volatility, but since all firms smooth, investors rationally "undo" the effect of smoothing, thereby leaving volatility perceptions unchanged. Knowing that this is the equilibrium outcome, however, does not make it privately optimal for any firm to avoid smoothing, since not smoothing when everybody else is smoothing is not the same as not smoothing when no one is smoothing.

An interesting question this raises is whether there are multiple equilibria in this model, one of which is an equilibrium in which no firm smooths. The following result says no.

PROPOSITION 4. For sufficiently low weight (α) assigned to the current price (P_1) in the manager's compensation, there is no Nash equilibrium in which the manager does not smooth reported earnings.

The intuition is as follows. If no firm smooths in equilibrium, then investors expect all firms to accurately report economic earnings. It would now pay for a firm with high earnings volatility to smooth reported earnings to fool investors into giving it a higher price than its value to shareholders. The restriction on α is simply to preclude earnings inflation in the report. Thus, it cannot be an equilibrium for all firms to eschew smoothing.

It is possible to visualize an alternative model in which there is partial pooling, so that a particular report is associated with many possible values of economic earnings. Investors will now be unable to perfectly distinguish between firms with smooth and volatile earnings. The firms with the most volatile earnings (bad type) will get their first-best outcome, whereas firms with truly smooth earnings (good type) will be worse off than in a world with no smoothing.

Earnings smoothing reduces the time-series volatility of reported earnings, but only in an expected sense. Sometimes smoothing will not lower volatility because the ability of the manager to smooth is limited. Since the manager can only shift earnings from one period to another, he cannot always report earnings that are higher (or lower) than economic earnings. If earnings are high in the first period, the manager reports lower earnings and tries to spread the positive shock over two periods. The manager must then report earnings higher than the economic earnings in the second period. If the second-period economic earnings turn out to be even higher than first-period earnings, the manager's report may end up increasing the measured time-series volatility of reported earnings. Thus, there can be instances where smoothing fails in hindsight. But the ex ante probability of such events is low with serially uncorrelated earnings shocks, and so smoothing does reduce the expected value of the time-series volatility of reported earnings.

We now justify the manager's compensation scheme. We assume that the manager's compensation contract is written by representatives of the shareholders and aligns the manager's interests with those of the shareholders. Although not crucial to our main result that earnings smoothing is an equilibrium, we assume that nonowners cannot observe the compensation contract.[18]

The manager's reporting strategy affects e_{r1} and e_{r2}, which, in turn, determine prices P_1 and P_2. Shareholders want to maximize their payoffs up to $t = 2$. Since we assume that no trading takes place before $t = 2.5$, the payoff of a shareholder holding a share until $t = 2$ is $e_{r1} + e_{r2} + P_2 = e_{a1} + e_{a2} + P_2$. The manager cannot influence e_{a1} or e_{a2}, and so shareholders want to maximize P_2 and write a compensation contract proportional to P_2, that is, set $\alpha = 0$.

We can relax the assumption of no trading between $t = 1$ and $t = 2$. Suppose that when the contract is written at $t = 0$, shareholders expect to sell π shares between $t = 1$ and $t = 2$. The payoff from selling a share in this period is $e_{r1} + P_1$, whereas the expected payoff from holding a share beyond $t = 2$ is $e_{r1} + e_{r2} + P_2 = e_{a1} + e_{a2} + P_2$. Therefore, shareholders write a compensation contract proportional to $\pi(e_{r1} + P_1) + (1 - \pi)P_2$. This is exactly the form we have in (9) with $\alpha = \pi$. Thus, if the fraction of shares expected to be sold between $t = 1$ and $t = 2$ is sufficiently small, the compensation contract written by the shareholders induces the manager to smooth earnings.

V. A Numerical Example

In this section, we illustrate the smoothing equilibrium with an example. The earnings in any period are $\mu + \delta$ with a positive shock (probability 0.5) and $\mu - \delta$ with a negative shock (probability 0.5). The earnings mean, μ, is \$4 or \$5, with probability 0.5 each, and the earnings volatility, δ, is \$2 or \$3, with probability 0.5 each. The manager can report earnings that differ from the economic earnings by, at most, \$1. The cost of acquiring a private signal is \$.20 for each investor. Investors have \$1.20 each, and so any investor getting informed is left with \$1.00 to invest. The dollar measure of liquidity

18. We are thinking of the representative of shareholders as the board of directors (who own stock) or a large "inside" shareholder. We can either assume that the contract they write for the manager is unobservable to the market (and hence to small shareholders and nonowners) or observable. We prefer the unobservability assumption because it corresponds to actual practice. In this case, the optimal contract involves $\alpha = 0$ and emerges as a unique equilibrium. The reason is as follows: suppose that outsiders believe that the manager's compensation depends only on P_2. In this case, they will expect him to smooth. Given this, it does not pay for the inside shareholders to make the manager's contract depend on $P_1 + e_{r1}$ because that reduces smoothing (completely eliminating it if the contract depends only on $P_1 + e_{r1}$) when the market expects smoothing; this will lower the stock price. By contrast, if outsiders believe that the manager's compensation depends only on $P_1 + e_{r1}$, it will pay for the inside shareholders to compensate the manager based on P_2 to induce him to smooth earnings since this will increase the stock price. Thus, paying the manager based only on P_2 and the associated smoothing is a unique equilibrium. If we assume that outside shareholders can observe the manager's contract, the inside shareholders realize that they cannot fool the market into believing that the manager is not smoothing when he is smoothing. Since smoothing does not affect the stock price when the market correctly anticipates it, multiple equilibria are possible. For example, in one equilibrium, the manager's compensation is tied to $P_1 + e_{r1}$, and there is no smoothing; in another, it is tied to P_2, and there is smoothing.

supply at $t = 2.5$ varies between 0 and 1. Liquidity supply l has a triangular distribution that peaks in the middle:

$$f(l) = \begin{cases} 4(0.5 - |l - 0.5|) & \text{if } 0 \leq l \leq 1 \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

Speculators who spend \$.20 to become informed at $t = 2.5$ observe their private signal about the earnings shock at $t = 3$ and submit their demand for shares only if earnings are to experience a positive shock. The market maker sees the sum of the flow of liquidity and speculator trades. He uses the aggregate order flow to form Bayesian beliefs about the nature of the shock to earnings in the next period and then sets a market-clearing price. The equilibrium measure of speculators, $\theta^*(\mu, \delta)$, is such that the expected profit from trading in the future equals \$.20. The expected profit, and hence the measure of speculators, depends on the characteristics of earnings. We numerically compute $\theta^*(\mu, \delta)$ for different μ and δ combinations:

$$\begin{aligned} \theta^*(5,2) &= 0, \\ \theta^*(5,3) &= 0.31163, \\ \theta^*(4,2) &= 0.21566, \\ \theta^*(4,3) &= 0.40824. \end{aligned} \tag{17}$$

This measure of speculators is increasing in the ratio of the earnings volatility to the earnings mean. Liquidity-seeking shareholders suffer average trading losses that equal the aggregate cost of information acquisition so that their expected trading losses equal the measure of speculators times the individual cost of information acquisition. The price at $t = 2$, which represents the expected future payoffs to the holder of a share, equals the mean of earnings minus expected trading losses. Denoting this function by $P^*(\mu, \delta)$, we have:

$$\begin{aligned} P^*(5,2) &= \$5, \\ P^*(5,3) &= \$4.938, \\ P^*(4,2) &= \$3.957, \\ P^*(4,3) &= \$3.918. \end{aligned} \tag{18}$$

We assume that the manager's objective is linearly increasing in the price at $t = 2$. The manager will report earnings smoothed to the maximum extent possible. The equilibrium smoothing strategy is defined as follows:

If first-period economic earnings experience a negative shock (\$3 or less), the manager reports earnings \$1 higher than the economic earnings.

If first-period economic earnings experience a positive shock (\$6 or more), the manager reports earnings \$1 lower than the economic earnings.

In the second period, the manager has no choice. He must report the aggregate of any outstanding "earnings balance" from the previous period and the second-period earnings. To show that the manager smooths in a sequential equilibrium, we specify the beliefs held by investors.

Beliefs of investors at $t = 1$:

If reported earnings in the first period are low ($4 or less), economic earnings are $1 lower than the reported earnings.

If reported earnings in the first period are high ($5 or more), economic earnings are $1 higher than the reported earnings.

Beliefs of investors at $t = 2$:

If reported earnings are high or low in both periods, the economic earnings in each period equal the average of the two reported earnings.

If reported earnings are high in one period and low in the other, then (i) the mean of economic earnings is the average of the two reported earnings, (ii) the economic earnings are $1 less than the lower reported earnings and $1 more than the higher reported earnings, and (iii) the volatility is the difference between the mean and the economic earnings in either of the two periods.

With these beliefs, investors set the price as the expected value of future payoffs. At $t = 2$, this price (ex-dividend) is given by the function in (18) if the mean and the volatility are both known and by a weighted average of these values if there is uncertainty about the mean and the volatility. Thus, if investors know that the sum of the mean and the volatility is $7, a mean of $5 and a volatility of $2 is as likely as a mean of $4 and a volatility of $3. In that case, the price at $t = 2$ is the average of $5 and $3.918, that is, $4.459.

The price at $t = 1$ is the sum of the expected reported earnings at $t = 2$ and the expected price at $t = 2$. Thus, if the manager reports earnings of $2 at $t = 1$, investors believe that economic earnings are $1 and that the manager has inflated his report by $1, a sum that he will have to offset in his report of next period's earnings. With $1 economic earnings, the mean must be $4 and the volatility must be $3. Therefore, the expected value of next period's reported earnings is \$4 − \$1 = \$3, and the expected price at $t = 2$ is $3.918. Therefore, the price at $t = 1$ is $6.918.

Given the beliefs of investors, the manager smooths earnings. To conserve space, we show this only for one case of high first-period earnings and one case of low first-period earnings.

Consider economic earnings of $7 in the first period. This means that earnings experienced a positive shock. As seen from the summary of investors' beliefs at $t = 2$ given above, investors will form correct beliefs about the mean of the earnings after observing the reported earnings in the two periods. The manager wants to minimize investors' estimate of volatility. If earnings are again high ($7) in the second period, investors will infer that the economic earnings were $7 in each period regardless of what the manager reports. The only case in which the manager's report may influence investor perceptions erroneously is when earnings are low in the second period. If the manager smooths as expected and reports \$6, investors will infer that economic earnings were \$7 in the first period, and they will form correct beliefs. If, however, the manager does not smooth, investors may sometimes overestimate volatility.

Suppose, for example, that the manager truthfully reports \$7. Investors will think that the economic earnings were actually \$8. Suppose that economic earnings in the second period are \$3. The manager reports \$3 truthfully, but this time the investors will think economic earnings were \$2. In this case, the investors correctly infer the mean to be \$5 but overestimate the volatility to be \$3 instead of the actual \$2. The price set in the second period will then be \$4.938, although it should have been \$5. The manager will smooth in order to maximize the second-period price.

Smoothing also occurs in the case of low earnings. Suppose that earnings in the first period are \$2. If the manager smooths and reports \$3, investors infer economic earnings correctly. If, however, the manager does not smooth, investors may overestimate volatility. Suppose that the manager chooses to truthfully report \$2. Investors then think that economic earnings were \$1. If economic earnings in the second period turn out to be \$6, the manager again truthfully reports \$6. This time investors think that the economic earnings were \$7. They then correctly infer the mean to be \$4 but overestimate the volatility to be \$3 instead of the true value of \$2. The price set at $t = 2$ is then \$3.918, whereas it should have been \$3.957. The manager is once again better off smoothing.

VI. Robustness and Qualifications

In this section, we discuss a variety of issues related to the robustness of our analysis. We also consider possible extensions.

A. Welfare Implications

The legal and accounting environment can affect the discretion available to the manager in reporting to financial markets and thereby influence the extent of earnings smoothing. What are the welfare implications of reporting discretion in the context of our model? Note, first, that information acquisition represents a welfare loss in our model. It simply consumes resources and redistributes wealth across investors; the improved transparency of prices due to informed trading has no welfare effect in our model. The fact that earnings smoothing by firms discourages welfare-reducing information acquisition by speculators suggests that allowing greater reporting discretion to managers might improve welfare. However, this is not the case, since smoothing does not reduce information acquisition as investors are not fooled in equilibrium. That is, the measure of speculators in equilibrium is unaffected by the degree of smoothing as long as reported earnings contain some information. Thus, our model does not provide any argument for or against increasing reporting discretion to managers.

However, it is true that any policy change that reduces the aggregate information acquisition cost improves welfare. For example, if an increase in the mandatory information disclosed by firms in addition to reported earnings were to reduce speculators' information acquisition costs (e.g., in Boot and Thakor, forthcoming), firms should be required to disclose information to the extent that the marginal cost of disclosing information equals the marginal aggregate cost of residual information acquisition by speculators acting privately.

B. Capital Structure Assumptions

We have assumed that the firm has no debt. One implication of introducing debt is that it could act as a signal of future cash flows (e.g., Ross 1977). However, in our model there is no informational asymmetry about future cash flows at $t = 0$, the time that the debt issue decision would presumably be made. The manager becomes privately informed only when he observes the economic earnings at $t = 1$. Thus, debt has no apparent signaling role in the usual sense.

There is another aspect of debt that could, however, affect the analysis. Boot and Thakor (1993) show that debt magnifies the volatility of shareholders' claims, encourages information acquisition, and thereby increases the expected trading losses of shareholders. Thus, keeping the volatility of earnings fixed, a higher debt-to-equity ratio increases the price discount of equity due to expected trading losses, strengthening the manager's smoothing propensity.[19]

C. Dividend Smoothing

Our rationale for earnings smoothing can be generalized to other measures of corporate performance, such as dividends.[20] In this model, reported earnings and dividends are identical. In a more general model with investment, earnings and dividends may differ. Earnings and dividends in such a model will each carry useful valuation-relevant information, with neither being a sufficient statistic for the other. In such a setting, our model could be interpreted as explaining both earnings and dividend smoothing. In fact, dividends can typically be smoothed more than earnings, and one does observe smoothing of dividends even relative to a (smoothed) reported earnings stream. This is consistent with the core intuition of our article.

D. Earnings Smoothing or Earnings Inflation?

Our manager smooths earnings to influence investors' perceptions about earnings volatility. We require that the uncertainty in the volatility of earnings be high relative to the uncertainty in the mean of earnings, as formally stated in

19. This result does depend on the assumption that current shareholders are the liquidity seekers.
20. We thank two anonymous referees for suggesting this general interpretation of our results.

assumption 4. This assumption is critical to our results when there is trading between dates 1 and 2.[21] If we make the opposite assumption that uncertainty in the mean is high relative to the uncertainty in the volatility, the manager may have an incentive to influence investors' beliefs about the mean rather than the volatility and would inflate earnings to signal a higher mean. Assumption 4 characterizes the conditions under which the manager's incentive to smooth earnings dominates the incentive to inflate earnings.

We suspect that, when the uncertainty in the mean is about the same as the uncertainty in the volatility, the manager combines earnings smoothing with earnings inflation. Thus, the manager reports inflated earnings when earnings are low and lower-than-actual earnings when earnings are high, but the manipulation is larger when the earnings are low than when they are high.

E. Acquisition of Private Information by Shareholders

An apparently strong assumption in our analysis is that shareholders are selling for liquidity reasons and none of them acquire private information. This simplifying assumption can be relaxed, however. The analysis uses the fact that the expected losses of shareholders due to liquidity trading are proportional to the information acquisition costs of the speculators. An alternative would be to allow some of the shareholders to get informed and to endogenously determine how many shareholders become informed. In such a model, the marginal benefit of getting informed equals the cost of information acquisition in equilibrium, taking into account the possibility of liquidity-motivated trading losses in the future. It is difficult to specify choice of information acquisition, trading, and pricing in this setting without imposing further structure. However, the analysis goes through even if some speculators are shareholders as long as all the liquidity seekers are also shareholders of the firm so that the information acquisition costs of all speculators are still paid by the shareholders. The key is that the measure of speculators should be increasing in earnings volatility; this seems likely because high earnings volatility increases the ex ante value of private information.

F. Buying and Selling by Liquidity Investors

We can also relax the restriction that all liquidity-motivated investors are sellers; we could allow some to be buyers and some of these buyers to be existing shareholders. In this case, the losses of liquidity trading will be borne by all these noise (liquidity-motivated) investors. What we need is the assumption that a person owning the shares of the firm is expected to lose more due to liquidity trading than a person not owning shares. In this case, what matters to current shareholders is the expected loss due to liquidity trading that is in excess of the expected loss due to liquidity trading if they did not own shares.

21. If there is no trading between dates 1 and 2, the manager's compensation contract depends only on the price at date 2, and our results do not require assumption 4.

G. *Cost of Smoothing*

We have assumed that the discretion exercised by the manager does not have any real effects on the firm's performance. In practice, there will be real costs of smoothing, such as the cost of manipulating accounts and the cost of suboptimal decisions. We have assumed that these costs are zero when the manager manipulates earnings by less than d and infinite when the manager manipulates earnings by more than d. We suspect that a continuous cost function will not alter the flavor of the results, although the extent of smoothing may be reduced.

H. *Benefits of Information Acquisition*

We did not assume any public benefits to information acquisition. When information acquisition results in spillover externalities that lead to better decisions within the firm, some of the benefits of information acquisition will also accrue to shareholders (see Allen 1993; Boot and Thakor 1997; Allen and Gale 1999). Earnings smoothing should be less for such firms.

I. *Extension to Multiple Periods*

It is natural to wonder how robust our results are when the model is extended from three periods to perpetuity. So we now provide a discussion of this issue. This requires that we make some simplifying assumptions. First, we assume that the mean is known with certainty and is common knowledge. As discussed earlier, uncertainty about the mean relative to the uncertainty about the volatility determines whether the manager smooths or inflates earnings. A known mean eliminates the manager's incentive to inflate earnings. We also modify the earnings distribution so that finitely many observations are not sufficient to completely infer the distribution. Assume that earnings in each period are independent and identically distributed. The distribution is normal with mean zero and an unknown variance. Everyone has common prior beliefs about the variance (volatility).

Only the manager sees the economic earnings, based on which he reports, with some discretion, to investors. He can report higher or lower earnings and carry a balance to the next period. This balance, either a surplus or a deficit, cannot exceed a fraction β of the current-period earnings. In the next period, the balance carried forward and the new earnings are available to the manager. The manager must now report total earnings that include the balance and at least a fraction $1 - \beta$ of the new earnings and at most a fraction $1 + \beta$ of the new earnings.

Investors rationally form beliefs about economic earnings by observing reported earnings. Some shareholders experience liquidity shocks in a future period, and this is followed by trading. Some investors can acquire a private signal about the firm's prospects before trading in that period; they do not buy such signals in prior periods as nobody would trade with them in the absence of liquidity shocks. The trading mechanism is exactly the same as in our three-period model.[22] It can be shown that the incentives for getting informed are increasing in the perceived volatility of the earnings stream. Thus, the higher the perceived volatility of the earnings distribution, the higher the measure of speculators and the higher the expected losses of liquidity-seeking shareholders, leading to a lower price of the shares. If the manager's objective function is some weighted average of prices in different periods, he wants to reduce perceived volatility in those periods. One should intuitively expect the manager to report smoothed earnings. We show that this is the case.

Define the equilibrium smoothing strategy as one in which the manager reports earnings equal to the sum of the balance from the previous period and a fraction $1 - \beta$ of the current period's earnings. Suppose that investors expect the manager to follow this strategy in every period.

The smoothing strategy is a one-to-one mapping from economic earnings to reported earnings. Further, investors and the manager agree that the previous balance in the first period is zero. If the manager smooths in the first period, investors infer the economic earnings as well as the true balance. Then, if the manager again smooths in the second period, investors again infer the economic earnings and the new balance in the second period. Thus, if the manager smooths consistently, investors infer the economic earnings in each period. The volatility they infer from reported earnings is an unbiased estimate of the true earnings volatility. With a normal distribution, it can be shown that the sum of the squared economic earnings is a sufficient statistic for estimating volatility. The higher this sum, the higher is the estimate of volatility.

Consider a deviation by the manager from the equilibrium smoothing strategy in any period. Suppose that, when the economic earnings in that period are positive, the manager reports higher earnings than those dictated by the smoothing strategy. Investors now infer earnings to be higher than economic earnings. If the economic earnings are negative, suppose that the manager reports lower earnings than dictated by the smoothing strategy. In this case, investors would infer earnings to be lower than economic earnings. Thus, investors perceive earnings to have a wider dispersion than that of economic earnings, which means that their estimate of volatility is biased upward. Further, we show that after such a deviation, this bias tends to increase in future periods.

Consider a period in which the balance from the previous period is B but investors believe it is C. First assume that $B > C$. Suppose that economic earnings are x and that the manager reports y. Investors expect the manager to smooth and report $C + (1 - \beta)z$ when economic earnings are z. Therefore, from the report y, they infer that economic earnings are $(y - C)/(1 - \beta)$. If the manager wants to minimize the magnitude of inferred earnings z, the best that he can do is to report a value as close as possible to C. With this strategy, the

22. We will need a positive mean so that the price is never negative for the trading mechanism to work. We are assuming zero mean just to simplify the expressions for smoothing strategy, and a positive mean will not change the results in any way.

manager's report and investors' beliefs are these:

$$y = B + x(1-\beta), z = \frac{B-C}{1-\beta} + x, B' = \beta x, C' = \beta z \quad \text{if } x > 0;$$

$$y = B + x(1+\beta), z = \frac{B-C}{1-\beta} + \frac{x(1+\beta)}{(1-\beta)}, B' = -\beta x, C' = \beta z \quad \text{if } \frac{C-B}{1+\beta} < x < 0;$$

$$y = C, z = 0, B' = B - C + x, C' = 0 \quad \text{if } \frac{C-B}{1-\beta} < x < \frac{C-B}{1+\beta};$$

$$y = B + x(1-\beta), z = \frac{B-C}{1-\beta} + x, B' = \beta x, C' = \beta z \quad \text{if } x < \frac{C-B}{1-\beta}.$$

Here B' is the balance carried to next period and C' is what investors think it is. We can show that for any value of B and C, the expected value of z^2 is more than the expected value of x^2 when x is normally distributed. We can also see that the investors' beliefs about next period's balance are also incorrect. Thus, even when the manager tries his best to reduce the magnitude of z, the signal about volatility from this period is biased upward.

We can similarly consider the case where investors' perception of the balance C is higher than the actual balance B. In this case, the manager can reduce investors' perception of economic earnings z using the following strategy:

$$y = B + x(1-\beta), z = \frac{B-C}{1-\beta} + x, B' = \beta x, C' = \beta z \quad \text{if } x > \frac{C-B}{1-\beta};$$

$$y = C, z = 0, B' = B - C + x, C' = 0 \quad \text{if } \frac{C-B}{1+\beta} < x < \frac{C-B}{1-\beta};$$

$$y = B + x(1+\beta), \ z = \frac{B-C}{1-\beta} + \frac{x(1+\beta)}{(1-\beta)}, B' = -\beta x, C' = \beta z \quad \text{if } 0 < x < \frac{C-B}{1+\beta};$$

$$y = B + x(1-\beta), z = \frac{B-C}{1-\beta} + x, B' = \beta x, C' = \beta z \quad \text{if } x < 0.$$

We can show in this case also that the expected value of z^2 is more than the expected value of x^2. The balance for next period is also different from what investors think. Thus, we see that, whenever investors' beliefs about the balance in a given period are incorrect, they overestimate the volatility in that period. The manager can thus minimize perceived volatility by smoothing.

VII. Conclusion

We have developed a model in which earnings smoothing is motivated by the desire to reduce the perceived volatility of the firm's earnings stream and discourage speculators from spending resources to acquire private information that could then be used to trade against shareholders selling for liquidity reasons. We can extract a few empirical implications from our analysis. Proposition 3 shows that the manager smooths earnings when his compensation is tied to P_2.[23] Thus, the first empirical implication is that a firm whose manager's compensation contract is tied to long-run performance is more likely to smooth earnings than a firm whose manager's compensation contract is tied to short-term performance. That is, somewhat surprisingly, earnings smoothing is just not something that arises from a preoccupation with short-term performance.

We have also argued that the manager's compensation contract should be tied to long-term performance.[24] This appears to be the case in practice. A major component of executive compensation these days is in the form of stock options that are given when the firm reports good results in a period but that are valuable only if the firm continues to perform well in future periods.

A second prediction of the analysis is that the degree of earnings smoothing will be higher for firms with higher uncertainty about the earnings volatility. In contrast, the managers of firms with high uncertainty about their earnings mean and low uncertainty about their earnings volatility are more likely to report inflated earnings.

A third prediction is that firms with large institutional ownership will smooth less because institutions are less likely to sell for liquidity reasons. When there are few liquidity-motivated shareholders, trading is driven largely by speculators, and the order flow reflects a lot of private information of speculators. The advantage of getting informed is low in such a setting, and few noninstitutional investors acquire costly information. Thus, if institutional investors have owned the firm's shares for a while and enjoy "incumbency informational advantages," the manager will not smooth earnings significantly. If the volatility of economic earnings is not correlated with the degree of institutional ownership, we should expect reported earnings to be more volatile for firms with large institutional holdings.[25] This, however, seems to be at odds with the existing evidence.[26] We suspect this is because the volatility of earnings is correlated with some other attribute (such as firm size) that is considered by institutional investors in choosing stocks.

It might, however, be better to test our prediction more directly. The prediction is that greater institutional ownership is associated with less earnings smoothing. Jiambalvo, Rajgopal, and Venkatachalam (2002) test this prediction and find supporting results.

A fourth prediction is that firms with more diffuse ownership structures—numerous shareholders with each owning a relatively small fraction of the firm—will smooth earnings more. This is because we would expect small shareholders to be more likely to sell their shares for liquidity reasons. This prediction can be tested with shareholder concentration data.

23. Prices formed beyond $t = 2$ do not enter the contract because the manager's actions do not directly affect them.

24. The compensation will be tied to short-term performance for firms with high uncertainty about mean and low uncertainty about the volatility.

25. We thank an anonymous referee for pointing this out.

26. See, e.g., Potter (1992).

Future research could be directed at examining the potential asset pricing implications of earnings smoothing. For instance, smoothing may affect the liquidity of the firm's stock at different points in time, and this could affect stock price dynamics.

Appendix

Proofs

We first prove an intermediate result.

LEMMA A1. Price is a decreasing function of net supply.

Proof. From (6),

$$P(S,\theta) = \frac{\frac{f(S+\theta)}{f(S)}(\hat{\mu}+\hat{\delta}) + (\hat{\mu}-\hat{\delta})}{\frac{f(S+\theta)}{f(S)}+1}$$

is increasing in $f(S+\theta)/f(S)$, and so it suffices to show that $f(S+\theta)/f(S)$ is decreasing in S. The first derivative of $f(S+\theta)/f(S)$ with respect to S is

$$\frac{f(S+\theta)}{f(S)}\left[\frac{f'(S+\theta)}{f(S+\theta)} - \frac{f'(S)}{f(S)}\right],$$

which is negative as $f'(x)/f(x)$, the slope of $\ln[f(x)]$, is decreasing in x if f is log-concave. Q.E.D.

Proof of lemma 1. We will show that

$$\frac{-\hat{\delta}f^*}{(\hat{\mu}+\hat{\delta})} < \frac{\partial}{\partial\theta}m(\hat{\mu},\hat{\delta},\theta) < 0, \tag{A1}$$

where $f^* = f(l^*)$. From (6) and (7),

$$\begin{aligned} m(\mu,\delta,\theta) &= \frac{\mu+\delta}{2}\int_{\underline{l}}^{\bar{l}} \frac{f(l)}{P(l-\theta,\theta)}dl - \frac{1}{2} \\ &= \frac{\mu+\delta}{2}\int_{\underline{l}}^{\bar{l}} \frac{f(l)+f(l-\theta)}{f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)}f(l)dl - \frac{1}{2}. \end{aligned}$$

Thus,

$$\frac{\partial}{\partial\theta}m(\mu,\delta,\theta) = \frac{\mu+\delta}{2}$$
$$\times \int_{\underline{l}}^{\bar{l}} \left(\frac{-f'(l-\theta)[f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)] + f'(l-\theta)(\mu-\delta)[f(l)+f(l-\theta)]}{[f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)]^2} \right)$$
$$\times f(l)dl \tag{A2}$$
$$= -(\mu+\delta)\delta \int_{\underline{l}}^{\bar{l}} \frac{f'(l-\theta)f^2(l)}{[f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)]^2}dl.$$

Let f achieve its maximum at l^*. Breaking the integral in (A2) into two parts, we get

$$\frac{\partial}{\partial\theta}m(\mu,\delta,\theta) = -(\mu+\delta)\delta(I_1+I_2),$$

where

$$I_1 = \int_{\underline{l}}^{l^*+\theta} \frac{f'(l-\theta)f^2(l)}{[f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)]^2}dl$$
$$= \int_{\underline{l}}^{l^*+\theta} \frac{f'(l-\theta)}{\left[(\mu+\delta)+\frac{f(l-\theta)}{f(l)}(\mu-\delta)\right]^2}dl$$
$$> \frac{1}{\left[(\mu+\delta)+\frac{f(l^*)}{f(l^*+\theta)}(\mu-\delta)\right]^2} \int_{\underline{l}}^{l^*+\theta} f'(l-\theta)dl$$

since log-concavity of f implies that $f'(l)>0$ for $l<l^*$ and $f(l-\theta)/f(l)$ is increasing in l (see proof of lemma A1 above). Moreover,

$$I_2 = \int_{l^*+\theta}^{\bar{l}} \frac{f'(l-\theta)f^2(l)}{[f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)]^2}dl$$
$$> \frac{1}{\left[(\mu+\delta)+\frac{f(l^*)}{f(l^*+\theta)}(\mu-\delta)\right]^2} \int_{l^*+\theta}^{\bar{l}} f'(l-\theta)dl$$

since log-concavity of f implies that $f'(l)<0$ for $l>l^*$ and $f(l-\theta)/f(l)$ is increasing in l. Thus,

$$\frac{\partial}{\partial\theta}m(\mu,\delta,\theta) < \frac{-(\mu+\delta)\delta}{\left[(\mu+\delta)+\frac{f(l^*)}{f(l^*+\theta)}(\mu-\delta)\right]^2}\int_{\underline{l}}^{\bar{l}} f'(l-\theta)dl$$

$$= \frac{-(\mu+\delta)\delta f(\bar{l}-\theta)}{\left[(\mu+\delta)+\frac{f(l^*)}{f(l^*+\theta)}(\mu-\delta)\right]^2} < 0.$$

To prove the other inequality, we have

$$I_1 = \int_{\underline{l}}^{l^*+\theta} \frac{f'(l-\theta)f^2(l)}{[f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)]^2}dl$$

$$< \int_{\underline{l}}^{l^*+\theta} \frac{f'(l-\theta)f^2(l)}{[f(l)(\mu+\delta)]^2}dl$$

$$= \frac{1}{(\mu+\delta)^2}\int_{\underline{l}}^{l^*+\theta} f'(l-\theta)dl = \frac{f(l^*)}{(\mu+\delta)^2},$$

and $I_2 < 0$. Thus,

$$\frac{\partial}{\partial\theta}m(\mu,\delta,\theta) > -(\mu+\delta)\delta\frac{f(l^*)}{(\mu+\delta)^2} = \frac{-\delta f^*}{\mu+\delta}.$$

Q.E.D.

Proof of proposition 1. We want to show that there is a unique value of θ^* satisfying the conditions of equilibrium in definition 1. Taking the supply and price functions as asserted, consider the expected trading profit m of each speculator as a function of θ. This is

$$m(\hat{\mu},\hat{\delta},0) = \frac{1}{2}\int_{\underline{l}}^{\bar{l}} \frac{\hat{\mu}+\hat{\delta}-P(l,0)}{P(l,0)}f(l)dl$$

$$= \frac{1}{2}\int_{\underline{l}}^{\bar{l}} \frac{\hat{\mu}+\hat{\delta}-\hat{\mu}}{\hat{\mu}}f(l)dl \quad \text{as } P(S,0) = \hat{\mu}$$

$$= \frac{\hat{\delta}}{2\hat{\mu}} > M, \text{ by assumption 2.}$$

Further, $m(.\ ,\ .\ ,\ \theta)$ is a strictly decreasing function of θ by lemma 1. Therefore, there is a unique $\theta^* > 0$ that satisfies the following condition for competition among speculators: $m(\hat{\mu}, \hat{\delta}, \theta^*) = M$. Q.E.D.

Proof of proposition 2.

a) From (6) and (7),

$$m(\mu,\delta,\theta) = \frac{\mu+\delta}{2}\int_{\underline{l}}^{\bar{l}} \frac{f(l)+f(l-\theta)}{f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)} f(l)dl - \frac{1}{2}$$

$$= \frac{1}{2}\int_{\underline{l}}^{\underline{l}+\theta} f(l)dl + \frac{1}{2}\int_{\underline{l}+\theta}^{\bar{l}} \frac{\frac{f(l)}{f(l-\theta)}+1}{\frac{f(l)}{f(l-\theta)}+\frac{\mu-\delta}{\mu+\delta}} f(l)dl - \frac{1}{2}. \tag{A3}$$

We see that $m(\mu,\delta,\theta)$ is decreasing in $(\mu-\delta)/(\mu+\delta)$, which is increasing in μ/δ.

b) From *a*, we see that $m(\mu,\delta,\theta)$ is increasing in δ/μ. We know from lemma 1 that m is decreasing in θ^*. So when δ/μ increases, θ^* must also increase to keep m unchanged at M, a condition for equilibrium. We now provide a lower bound on sensitivity of measure θ^* to volatility δ for a later result:

$$\frac{\partial}{\partial\delta}\theta^*(\mu,\delta) > \frac{\mu M}{\delta^2 f^*}, \tag{A4}$$

To prove (A4), we first derive the following lower bound for the sensitivity of expected trading profit to volatility.

$$\frac{\partial}{\partial\delta} m(\mu,\delta,\theta) > \frac{\mu M}{\delta(\mu+\delta)}. \tag{A5}$$

From (A3),

$$\frac{\partial}{\partial\delta} m(\mu,\delta,\theta) = \frac{1}{2}\int_{\underline{l}+\theta}^{\bar{l}} \frac{\partial}{\partial\delta}\left[\frac{\frac{f(l)}{f(l-\theta)}+1}{\frac{f(l)}{f(l-\theta)}+\frac{\mu-\delta}{\mu+\delta}}\right] f(l)dl$$

$$= \frac{\mu}{(\mu+\delta)^2}\int_{\underline{l}+\theta}^{\bar{l}} \left\{\frac{\frac{f(l)}{f(l-\theta)}+1}{\left[\frac{f(l)}{f(l-\theta)}+\frac{\mu-\delta}{\mu+\delta}\right]^2}\right\} f(l)dl$$

$$> \frac{\mu}{(\mu+\delta)^2}\int_{\underline{l}+\theta}^{\bar{l}} \frac{1}{\frac{f(l)}{f(l-\theta)}+\frac{\mu-\delta}{\mu+\delta}} f(l)dl$$

$$= \frac{\mu}{(\mu+\delta)}\int_{\underline{l}+\theta}^{\bar{l}} \frac{f(l-\theta)}{f(l)(\mu+\delta)+f(l-\theta)(\mu-\delta)} f(l)dl$$

$$= \frac{\mu}{\delta(\mu+\delta)} m(\mu,\delta,\theta) = \frac{\mu M}{\delta(\mu+\delta)}.$$

This proves (A5). To get (A4), note that in equilibrium, the measure of speculators is such that the profit of marginal speculator is M, a constant. This implies

$$\frac{\partial}{\partial\delta}\theta^*(\mu,\delta) = \frac{-\frac{\partial}{\partial\delta}m(\mu,\delta,\theta^*)}{\frac{\partial}{\partial\theta^*}m(\mu,\delta,\theta^*)} > \frac{-\frac{\mu M}{\delta(\mu+\delta)}}{\frac{-\delta f^*}{(\mu+\delta)}} = \frac{\mu M}{\delta^2 f^*},$$

using equations (A1) and (A5). Q.E.D.

Proof of lemma 2. The unconditional value of the firm is μ. The expected value of the future payoffs to the shareholders is, however, lower, and the difference constitutes the expected value of losses suffered by liquidity seekers. When liquidity-induced supply is l, shareholders get \$$l$ at $t = 2.5$ by selling some shares at market price, and they get a liquidating dividend for the remaining shares at $t = 3$. The market price at $t = 2.5$ depends on whether speculators demand shares or not and is $P(l - \theta^*, \theta^*)$ when third-period earnings experience positive shock and $P(l, \theta^*)$ when third-period earnings experience negative shock. The liquidating dividend per share is $\mu + \delta$ in the former case and $\mu - \delta$ in the latter. The expected losses L are thus given by

$$\begin{aligned} L &= \mu - \frac{1}{2}\int\left\{l + \left[1 - \frac{l}{P(l)}\right](\mu-\delta)\right\}f(l)dl - \frac{1}{2}\int\left\{l + \left[1 - \frac{l}{P(l-\theta^*)}\right](\mu+\delta)\right\}f(l)dl \\ &= -\frac{1}{2}\int\left[l - \frac{l(\mu-\delta)}{P(l)}\right]f(l)dl - \frac{1}{2}\int\left[l - \frac{l(\mu+\delta)}{P(l-\theta^*)}\right]f(l)dl. \end{aligned}$$

Thus,

$$\begin{aligned} -2L &= \int\left[(l-\theta^*) - \frac{(l-\theta^*)(\mu-\delta)}{P(l-\theta^*)}\right]f(l-\theta^*)dl \\ &\quad + \int\left[l - \frac{(l-\theta^*)(\mu+\delta)}{P(l-\theta^*)}\right]f(l)dl - \theta^*\int\frac{\mu+\delta}{P(l-\theta^*)}f(l)dl \\ &= \int(l-\theta^*)f(l-\theta^*)dl + \int lf(l)dl \\ &\quad - \int(l-\theta^*)\frac{(\mu-\delta)f(l-\theta^*) + (\mu+\delta)f(l)}{P(l-\theta^*)}dl - \theta^*\int\frac{\mu+\delta}{P(l-\theta^*)}f(l)dl \\ &= \int(l-\theta^*)f(l-\theta^*)dl + \int lf(l)dl \\ &\quad - \int(l-\theta^*)[f(l-\theta^*) + f(l)]dl - \theta^*\int\frac{\mu+\delta}{P(l-\theta^*)}f(l)dl \\ &= \theta^*\int\left[1 - \frac{\mu+\delta}{P(l-\theta^*)}\right]f(l)dl \\ &= -2\theta^* m(\mu,\delta,\theta^*). \end{aligned}$$

The third equality is obtained by substituting for price function from equation (6), and the last equality uses equation (7). Since $m(\mu,\delta,\theta^*) = M$ in equilibrium, we get $L = M\theta^*$, the cost of information acquisition by speculators. Q.E.D.

Proof of proposition 3. We will first show that the beliefs of the manager and

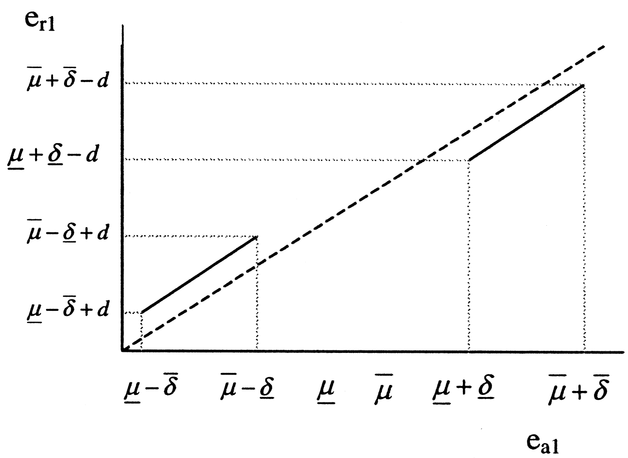

FIG. A1.—Smoothing strategy

the investors are rational given their strategies and then show that their strategies are incentive compatible.

Manager's beliefs: The manager observes e_{a1} at $t = 1$ and e_{a2} at $t = 2$ and can correctly infer λ_1 at $t = 1$ and λ_2 at $t = 2$ using (12). With his knowledge of e_{at} and λ_t, he forms Bayesian beliefs about μ and δ, using (1) and (3).

Investors' beliefs: Investors observe only the reported earnings e_{rt}, yet they can also correctly infer the nature of the shock to earnings, that is, λ_1 and λ_2 at $t = 1$ and $t = 2$, respectively, using (11) and (13). The manager's strategy at $t = 1$ (in accordance with [14]) is shown in figure A1. When investors observe an equilibrium earnings report, they form beliefs in accordance with the manager's strategy and infer the following:

If $\lambda_1 =$ high, then $e_{a1} = e_{r1} + d$ (note: $e_{r1} + d \leq \bar{\mu} + \bar{\delta}$ for equilibrium e_{r1});

if $\lambda_1 =$ low, then $e_{a1} = e_{r1} - d$ (note: $e_{r1} - d \geq \underline{\mu} - \bar{\delta}$ for equilibrium e_{r1}).

When investors observe an off-equilibrium earnings report, the sequential equilibrium concept allows them to hold any plausible beliefs about e_{a1} (see Kreps and Wilson 1982), which can lead to observed reported earnings. One possible off-equilibrium earnings report is $e_{r1} \geq \bar{\mu} + \delta - d$. Here reported earnings are greater than the maximum possible in equilibrium. Investors hold the plausible belief that the economic earnings are at the highest level possible, that is, $e_{a1} = \bar{\mu} + \delta$.

Another possible off-equilibrium earnings report is $e_{r1} \leq \underline{\mu} - \bar{\delta} + d$. Here reported earnings are lower than the minimum possible in equilibrium. Investors hold the plausible belief that the economic earnings are at the lowest level possible, that is, $e_{a1} = \underline{\mu} - \delta$.

Having formed beliefs about λ_1 and e_{a1}, investors must use (1) and (3) to form Bayesian beliefs about μ and δ. At $t = 2$, investors can correctly infer λ_1 and λ_2 using (11) and (13).

If $\lambda_1 = \lambda_2$, then $e_{a1} = e_{a2}$. Now, from (10), we get $e_{a1} = e_{a2} = (e_{r1} + e_{r2})/2$. Once investors know e_{a1} and e_{a2}, they have all the information the manager has, and they form Bayesian beliefs about μ and δ using (1) and (3).

If λ_1 and λ_2 are different, economic earnings are $\mu + \delta$ in one period and $\mu - \delta$ in the other. Therefore, the sum of earnings is 2μ. Investors use (10) to correctly infer $\mu = (e_{r1} + e_{r2})/2$.

Thus, at $t = 2$ investors always have the same beliefs about μ as the manager. They form rational beliefs about δ. If λ_1 is high, the manager's equilibrium strategy is $e_{r1} = e_{a1} - d = \mu + \delta - d$, and so investors use the rule $\delta = e_{r1} + d - \mu$. If this value of δ is too high to be feasible, investors infer that the manager did not smooth as expected and reported higher than what he should have in equilibrium. In this case, they assume that the economic earnings are the highest feasible value conditional on μ and that δ is the highest possible value. Thus, investors believe that $\delta = \min(e_{r1} + d - \mu, \bar{\delta})$.

If λ_1 is low, the manager's equilibrium strategy is $e_{r1} = e_{a1} + d = \mu - \delta + d$, and so investors use the rule $\delta = \mu - e_{r1} + d$. If this value of δ is too high to be feasible, investors infer that the manager did not smooth as expected and reported lower than what he should have in equilibrium. In this case, they assume that the economic earnings are the lowest feasible value conditional on μ and that δ is the highest possible value. Thus, investors believe that $\delta = \min(\mu - e_{r1} + d, \bar{\delta})$.

Investors' strategy: The investors form rational expectations about future earnings using (1) and (2) with their beliefs about μ and δ. The price set by investors in (15) is the competitive price. Using the fact that expected value of future earnings is u, we can simplify (15) to

$$P_1 = \begin{cases} \hat{E}^1[3\mu + \delta - M\theta^*(\hat{E}^2[\mu], \hat{E}^2[\delta])] - e_{r1} & \text{if } \lambda_1 = \text{high} \\ \hat{E}^1[3\mu - \delta - M\theta^*(\hat{E}^2[\mu], \hat{E}^2[\delta])] - e_{r1} & \text{if } \lambda_1 = \text{low}, \end{cases}$$

$$P_2 = \hat{E}^2[\mu] - M\theta^*(\hat{E}^2[\mu], \hat{E}^2[\delta]). \tag{A6}$$

Manager's strategy: The manager has no choice in his second-period report. For the first period, the manager's problem is to choose

$$e_{r1} = \arg \max_{x \in [e_{a1} - d,\, e_{a1} + d]} Z(x, e_{a1}),$$

where the objective function Z is

$$Z(x, y) = E[\alpha(e_{r1} + P_1) + (1 - \alpha)P_2 | e_{r1} = x, e_{a1} = y].$$

We first consider the case of a positive earnings shock in the first period, that is, $\lambda_1 =$ high. We shall analyze the two parts in $Z(x, y)$ separately. First, we consider the value of shares at $t = 1$:

$$\begin{aligned} e_{r1} + P_1 &= \hat{E}^1[3\mu + \delta - M\theta^*(\hat{E}^2[\mu], \hat{E}^2[\delta])] \\ &= E[\mu + \delta | e_{r1} = x] + 2E[\mu | e_{r1} = x] - ME[\theta^*(\hat{E}^2[\mu], \hat{E}^2[\delta]) | e_{r1} = x] \\ &= E[e_{a1} | e_{r1} = x] + 2E[\mu | e_{r1} = x] - ME[\theta^*(\hat{E}^2[\mu], \hat{E}^2[\delta]) | e_{r1} = x]. \end{aligned}$$

At $t = 1$, investors believe that $e_{a1} = \min(e_{r1} + d,\ \bar{\mu} + \bar{\delta})$. If there is another positive shock to earnings in the second period, investors will know e_{a1} but not the exact values of μ and δ. The expected values of μ and δ will be used to determine expected losses due to liquidity selling. On the other hand, if earnings experience a negative shock in the second period, investors will infer values of μ and δ, which will determine the expected losses due to liquidity selling. Therefore,

$$\begin{aligned} e_{r1} + P_1 = {} & \min(x + d, \bar{\mu} + \bar{\delta}) + 2E[\mu \mid \mu + \delta = \min(x + d, \bar{\mu} + \bar{\delta})] \\ & - \frac{M}{2}\theta^*\{E[\mu \mid \mu + \delta = \min(x + d, \bar{\mu} + \bar{\delta})], \\ & E[\delta \mid \mu + \delta = \min(x + d, \bar{\mu} + \bar{\delta})]\} \\ & - \frac{M}{2}E[\theta^*(\mu, \delta) \mid \mu + \delta = \min(x + d, \bar{\mu} + \bar{\delta})]. \end{aligned} \tag{A7}$$

The above expression becomes independent of x for $x > \bar{\mu} + \bar{\delta} - d$. For lower values of x,

$$\begin{aligned} & \frac{\partial}{\partial x}E[e_{r1} + P_1 \mid e_{r1} = x, e_{a1} = y] \\ & = 1 + 2\frac{d}{dx}E[\mu \mid \mu + \delta = x + d] \\ & \quad - \frac{M}{2}\frac{d}{dx}\theta^*(E[\mu \mid \mu + \delta = x + d], E[\delta \mid \mu + \delta = x + d]) \\ & \quad - \frac{M}{2}\frac{d}{dx}E[\theta^*(\mu, \delta) \mid \mu + \delta = x + d] \\ & \le 3 - \frac{M}{2}\frac{d}{dx}\theta^*(E[\mu \mid \mu + \delta = x + d], E[\delta \mid \mu + \delta = x + d]) \\ & \quad - \frac{M}{2}\frac{d}{dx}E[\theta^*(\mu, \delta) \mid \mu + \delta = x + d] \le 3. \end{aligned}$$

The first inequality follows from assumption 4*a*. The second inequality follows from assumption 4*a* and 4*b* and proposition 2*b*. Thus,

$$\frac{\partial}{\partial x}E[e_{r1} + P_1 \mid e_{r1} = x, e_{a1} = y]\begin{cases} = 0 & \text{if } x > \bar{\mu} + \bar{\delta} - d \\ < 3 & \text{if } x < \bar{\mu} + \bar{\delta} - d. \end{cases} \tag{A8}$$

Now, we consider the expected value of price at $t = 2$:

$$\begin{aligned} & E[P_2 \mid e_{r1} = x, e_{a1} = y] \\ & = E[\hat{E}^2[\mu] - M\theta^*(\hat{E}^2[\mu], \hat{E}^2[\delta]) \mid e_{r1} = x, e_{a1} = y]. \end{aligned} \tag{A9}$$

We saw earlier that at $t = 2$ investors have the same beliefs about μ as the manager. Therefore, $E[\hat{E}^2[\mu] \mid e_{r1} = x, e_{a1} = y] = E[\mu \mid e_{a1} = y]$.

If earnings experience another positive shock in the second period, investors will infer e_{a1} and calculate correct expected values of μ and δ. If earnings experience a negative shock in the second period, investors will infer μ and assume $\delta = \min(e_{r1} + d - \mu, \bar{\delta})$. Thus,

$$\begin{aligned} &E[P_2|e_{r1} = x, e_{a1} = y] \\ &= E[\mu|e_{a1} = y] - \frac{M}{2}\theta^*(E[\mu|e_{a1} = y], E[\delta|e_{a1} = y]) \qquad \text{(A10)} \\ &\quad - \frac{M}{2}E[\theta^*(\mu, \min(x + d - \mu, \bar{\delta}))|e_{a1} = y]. \end{aligned}$$

We see from equations (A7) and (A10) that the manager's objective function is independent of x for $x \geq \bar{\mu} + \delta - d$. Therefore, we only need to compare reporting strategies with $x \leq \bar{\mu} + \delta - d$. We will now show that, when the economic earnings are y, the manager prefers to report $y - d$ rather than any other report x with $|y - x| \leq d$ and $x \leq \bar{\mu} + \delta - d$. Consider the incremental benefit to the manager of reporting x rather than following the equilibrium strategy:

$$\begin{aligned} Z(x,y) - Z(y-d,y) = {}& \alpha\{E[e_{r1} + P_1|e_{r1} = x, e_{a1} = y] - E[e_{r1} + P_1|e_{r1} = y - d, e_{a1} = y]\} \\ &+ (1-\alpha)\{E[P_2|e_{r1} = x, e_{a1} = y] - E[P_2|e_{r1} = y - d, e_{a1} = y]\}. \end{aligned}$$

Using equations (A8) and (A10), we get

$$\begin{aligned} Z(x,y) - Z(y-d,y) \leq {}& 3\alpha\{x - (y - d)\} - (1-\alpha)\frac{M}{2}\{E[\theta^*(\mu, \min(x + d - \mu, \bar{\delta}))|e_{a1} = y] \\ &- E[\theta^*(\mu, \min(y - \mu, \bar{\delta}))|e_{a1} = y]\}. \end{aligned}$$

In the above expression, $y - \mu = e_{a1} - \mu = \delta \leq \bar{\delta}$. Thus,

$$\begin{aligned} Z(x,y) - Z(y-d,y) \leq {}& 3\alpha(x - y + d) \\ &- (1-\alpha)\frac{M}{2}\{E[\theta^*(\mu, \min(x + d - \mu, \bar{\delta}))|e_{a1} = y] - E[\theta^*(\mu, \delta)|e_{a1} = y]\} \\ \leq {}& 3\alpha(x - y + d) - \frac{(1-\alpha)\underline{\mu}M^2}{2f^*\delta^2}E[\min(x + d - \mu, \bar{\delta}) - \delta|e_{a1} = y]. \end{aligned} \qquad \text{(A11)}$$

The last inequality uses equation (A4). Now we analyze the last term in the above expression.

$$\begin{aligned} &E[\min(x + d - \mu, \bar{\delta}) - \delta|e_{a1} = y] \\ &= E[x + d - \mu - \delta|x + d - \mu < \bar{\delta}, e_{a1} = y]P(x + d - \mu < \bar{\delta}|e_{a1} = y) \\ &\quad - E[\bar{\delta} - \delta|x + d - \mu \geq \bar{\delta}, e_{a1} = y]P(x + d - \mu \geq \bar{\delta}|e_{a1} = y). \end{aligned}$$

Let us denote by r the excess of reported earnings x over the equilibrium report $y - d$. That is, $r = x - y + d$. We know $0 \leq r \leq 2d$ because $|y - x| \leq d$. Also denote by ϕ the probability $P(x + d - \mu \geq \bar{\delta} | e_{a1} = y)$. Then,

$$E[\min(x + d - \mu, \bar{\delta}) - \delta | e_{a1} = y]$$

$$= (x + d - y)(1 - \phi) + (\bar{\delta} - E[\delta | x + d - \mu \geq \bar{\delta}, \mu + \delta = x + d - r])\phi \qquad \text{(A12)}$$

$$= r(1 - \phi) + (\bar{\delta} - E[\delta | \delta \geq \bar{\delta} - r, \mu + \delta = x + d - r])\phi.$$

But,

$$x + d - r \leq (\bar{\mu} + \bar{\delta} - d) + d - r = \bar{\mu} + \bar{\delta} - r,$$

so we can use assumption 4*c* to get $E[\delta | \delta \geq \bar{\delta} - r, e_{a1} = y] \leq \bar{\delta} - kr$. Substituting in (A12) yields

$$E[\min(x + d - \mu, \bar{\delta}) - \delta | e_{a1} = y] \geq r(1 - \phi) + kr\phi = r\{1 - (1 - k)\phi\} \geq kr.$$

Using the above inequality and (A11), we get:

$$Z(x, y) - Z(y - d, y) \leq 3\alpha r - \frac{(1 - \alpha)\underline{\mu} M^2 kr}{2f^* \bar{\delta}^2}$$

$$\leq 0 \text{ if } \alpha \leq \alpha^* \equiv \frac{\underline{\mu} M^2 kr}{\underline{\mu} M^2 k + 6f^* \bar{\delta}^2}.$$

Therefore, if $\alpha < \alpha^*$, the manager prefers to report $e_{a1} - d$ rather than x. This means that the manager's strategy is incentive compatible when first-period earnings experience a positive shock.

We now consider the case of a negative shock to first-period earnings. We again analyze the values of shares at $t = 1$ and $t = 2$ separately:

$$e_{r1} + P_1 = \hat{E}^1[3\mu - \delta - M\theta^*(\hat{E}^2[\mu], \hat{E}^2[\delta])]$$

$$= E[\mu - \delta | e_{r1} = x] + 2E[\mu | e_{r1} = x] - ME[\theta^*(\hat{E}^2[\mu], \hat{E}^2[\delta]) | e_{r1} = x]$$

$$= E[e_{a1} | e_{r1} = x] + 2E[\mu | e_{r1} = x] - ME[\theta^*(\hat{E}^2[\mu], \hat{E}^2[\delta]) | e_{r1} = x].$$

At $t = 1$, investors believe that $e_{a1} = \max(e_{r1} - d, \underline{\mu} - \bar{\delta})$. If there is another negative shock in the second period, they expect to know nothing more, and expected loss due to liquidity selling will be determined by expected values of μ and δ conditional on e_{a1}. If there is a positive shock in the second period, investors expect to infer exact values of μ and δ. Thus,

$$e_{r1} + P_1 = \max(x - d, \underline{\mu} - \bar{\delta}) + 2E\mu | \mu - \delta = \max(x - d, \underline{\mu} - \bar{\delta})$$

$$- \frac{M}{2}\theta^*(E[\mu | \mu - \delta = \max(x - d, \underline{\mu} - \bar{\delta})], E[\delta | \mu - \delta = \max(x - d, \underline{\mu} - \bar{\delta})])$$

$$- \frac{M}{2}E[\theta^*(\mu, \delta) | \mu - \delta = \max(x - d, \underline{\mu} - \bar{\delta})].$$

In the above expression, the first term is clearly increasing in x. The second term is also increasing in x because of assumption 4*d*. The third term is again increasing in

x because of assumption 4d and proposition 2b. The fourth term is increasing in x because of assumption 4b. Thus, $e_{r1} + P_1$ is clearly increasing in x.

Now we consider the expected value at $t = 1$ of the price at $t = 2$:

$$E[P_2|e_{r1} = x, e_{a1} = y] = E[\hat{E}^2[\mu] - M\theta^*(\hat{E}^2[\mu], \hat{E}^2[\delta])|e_{r1} = x, e_{a1} = y]. \quad \text{(A13)}$$

But, as we saw earlier, $E[\hat{E}^2[\mu]|e_{r1} = x, e_{a1} = y] = E[\mu|e_{a1} = y]$. Further, investors' beliefs about δ at $t = 2$ match the manager's beliefs if earnings experience another negative shock in the second period. If earnings experience a positive shock in the second period, investors believe $\delta = \min(\mu - e_{r1} + d, \bar{\delta})$. Thus,

$$\begin{aligned} E[P_2|e_{r1} = x, e_{a1} = y] = E[\mu|e_{a1} = y] & \\ & - \frac{M}{2}\theta^*(E[\mu|e_{a1} = y], E[\delta|e_{a1} = y]) \\ & - \frac{M}{2}E[\theta^*(\mu, \min(\mu - x + d, \bar{\delta}))|e_{a1} = y]. \end{aligned}$$

In the above expression, the first two terms are independent of x, while the third term is increasing in x because of proposition 2b. Thus, $E[P_2|e_{r1} = x, e_{a1} = y]$ is increasing in x. The manager's expected compensation is an increasing function of $E[e_{r1} + P_1|e_{r1} = x, e_{a1} = y]$ and $E[P_2|e_{r1} = x, e_{a1} = y]$, both increasing in x. Therefore, the manager reports the highest possible earnings, that is, $x = y + d$. Thus, the manager's strategy is incentive compatible in the case of a negative shock to earnings in the first period. Q.E.D.

Proof of proposition 4. Suppose that there is an equilibrium in which the manager reports truthfully (no smoothing). Then investors believe that the economic earnings are the same as reported earnings. Consider a negative shock to earnings in the first period. If the manager reports earnings slightly higher than economic earnings, it increases the price at $t = 1$ as well as the expected price at $t = 2$, thereby increasing the manager's expected compensation. At $t = 2$, investors will be able to determine the mean μ correctly, whereas they may underestimate volatility due to smoothing by the manager. Since perceived volatility adversely affects price as discussed in Section IV, smoothing increases the expected price at $t = 2$. At $t = 1$, smoothing increases investors' perception about the mean and reduces their perception about the volatility. Both effects make the shares more attractive to investors, and thus smoothing unambiguously increases the price at $t = 1$.

Now consider a positive shock to earnings. Suppose that economic earnings are $y > \bar{\mu} + \underline{\delta}$, so that the volatility is clearly above its lowest bound. Consider the effect of reporting x, slightly smaller than y, on the manager's expected compensation. If the firm experiences a negative shock next period, the manager will report higher than economic earnings and thus would be able to fool investors by reducing the perceived volatility of the firm. This increases the expected price of shares at $t = 2$. Smoothing reduces the perceived mean μ as well as the perceived volatility δ at $t = 1$. These two effects influence the price at $t = 1$ in opposite directions. The expressions for the prices at $t = 1$ and $t = 2$ are given in (A7) and (A10), respectively, when investors expect the manager to smooth by d. We cancel terms with d because investors expect the manager to report truthfully, and we ignore boundary conditions to get the manager's objective as

$$\alpha\Big\{x + 2E[\mu|\mu + \delta = x] - \frac{M}{2}\theta^*(E[\mu|\mu + \delta = x], E[\delta|\mu + \delta = x])$$

$$- \frac{M}{2}E[\theta^*(\mu, \delta)|\mu + \delta = x]\Big\}$$

$$+ (1 - \alpha)\Big\{E[\mu|\mu + \delta = y] - \frac{M}{2}\theta^*(E[\mu|\mu + \delta = y], E[\delta|\mu + \delta = y])$$

$$- \frac{M}{2}E[\theta^*(\mu, x - \mu)|\mu + \delta = y]\Big\}.$$

Differentiating the above expression with respect to x and using assumptions 4*a* and 4*b*, proposition 2*b*, and (A4), the slope of the manager's objective with respect to the reported earnings x is at most

$$3\alpha - \frac{(1 - \alpha)M}{2}\frac{\underline{\mu}M}{\bar{\delta}^2 f^*},$$

which is negative if

$$\alpha < \frac{\underline{\mu}M^2}{\underline{\mu}M^2 + 6f^*\bar{\delta}^2}.$$

Thus, the manager strictly prefers to smooth if α is sufficiently small. When investors expect truthful reporting, the shareholders of the firm have an incentive to structure the manager's compensation with a small enough α to induce smoothing, discourage informed acquisition and thus reduce future trading losses when they experience liquidity shocks. Q.E.D.

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