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J. Finan. Intermediation

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## Correlated leverage and its ramifications<sup>☆</sup>



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### ARTICLE INFO

#### Article history:

Received 30 March 2012

Available online 21 September 2014

### ABSTRACT

This paper develops a theory in which housing prices, the capital structures of banks (mortgage lenders) and the capital structures of mortgage borrowers are all endogenously determined in equilibrium. There are four main results. First, leverage is a “positively correlated” phenomenon in that high leverage among borrowers is positively correlated with high leverage among banks, and higher house prices lead to higher leverage for both. The intuition is that first-time homebuyers with fixed wealth endowments must borrow more to buy more expensive homes, whereas higher current house prices rationally imply higher expected future house prices and therefore higher collateral values on bank loans, inducing banks to be more highly levered. Second, higher bank leverage leads to greater house price volatility in response to shocks to fundamental house values. Third, a bank’s exposure to credit risk depends not only on its own leverage but also on the leverage decisions of *other* banks. Fourth, positive fundamental shocks to house prices dilute

<sup>☆</sup> For their helpful comments, we thank anonymous referees, Viral Acharya (the editor), Sudipto Bhattacharya, Ulf von Lilienfeld-Toal, Robert Marquez (discussant), Robert McDonald (discussant), Darius Palia, Matt Pritsker (discussant), Peter Raupach (discussant), S. Viswanathan (discussant), and participants at the Banco de Portugal Conference on Financial Intermediation (June 2009), the NBER Summer Institute (July 2009), the Deutsche Bundesbank/Imperial College Conference on the Future of Banking Regulation (September 2009), the Financial Intermediation Research Society (FIRS) conference (June 2010), the CAREFIN/Bocconi Banking Conference on Matching Stability and Performance (September 2010), the NY Fed/NYU Stern Conference on Financial Intermediation (November 2010), the American Economic Association (AEA) conference (January 2011), the Western Finance Association (WFA) conference (June 2011), and seminars at Washington University in St. Louis, Federal Reserve Bank of New York, Federal Reserve Bank of San Francisco, Federal Reserve Bank of Chicago, Imperial College London, and Georgia Institute of Technology. Song acknowledges financial support from Smeal College of Business at Pennsylvania State University, and the hospitality of Singapore Management University where part of this work was done.

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financial intermediation by reducing banks' pre-lending screening, and this reduction in bank screening further increases house prices. Empirical and policy implications of the analysis are drawn out, and empirical evidence is provided for the first two main results. The key policy implications are that greater geographic diversification by banks, tying mortgage tax exemptions to the duration of home ownership, and increasing bank capital requirements when borrower leverage is high can help reduce house price volatility.

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## 1. Introduction

High bank leverage makes the financial system more fragile and crises more likely (e.g., [Allen and Gale \(2008\)](#)).<sup>1</sup> Indeed, prudential bank capital regulation is based on this fundamental premise (e.g., [Bhattacharya and Thakor \(1993\)](#) and [Freixas and Rochet \(1997\)](#)). The subprime crisis of 2007–09 is a striking example of the alacrity with which a high-leverage financial system can find itself beset with a crisis that further erodes capital and engenders forces that exacerbate the crisis.

However, our knowledge of the dynamics of financial-system leverage is limited. We do not know what causes banks to become more highly levered, *outside* of crises periods in which exogenous shocks impose losses on banks and drain capital. In other words, if more highly levered banks make the financial system more fragile, what causes banks to be so? A related issue is that consumer (borrower) leverage also increased substantially prior to the recent crisis (e.g., [Gerardi et al. \(2008\)](#)), and this may have been a significant contributing factor to the crisis (e.g., [Mian and Sufi \(2010, 2011\)](#)).<sup>2</sup> Was this higher consumer leverage just a coincidence or was it in any way related to bank leverage? What are the consequences of this?

We address these questions by developing a theoretical model that explores the relationship between the leverage decisions of borrowers and banks, in the context of the home mortgage market. We consider a two-period economy in which first-period home buyers with limited wealth endowments need bank loans to finance house purchases. Borrowers' leverage decisions are driven by first-period house prices that dictate the amounts they need to borrow. In any period, the equilibrium house price is determined by two factors: a fundamental housing-market shock which affects the utility a consumer attaches to home ownership, and the price and availability of credit from banks to buy houses. Higher house prices necessitate larger bank loans for consumers/borrowers and thus higher leverage for borrowers with fixed initial wealth endowments.<sup>3</sup> Since first-period house prices depend on *expected* house prices in the second period, banks (correctly) interpret high first-period prices as implying a low likelihood of low second-period house prices. This, in turn, lowers their assessment of the default probability on loans because borrowers repay with proceeds from selling houses to second-period buyers. So banks keep lower capital in the first period when first-period house prices are higher. This phenomenon, whereby the leverage ratios of

<sup>1</sup> There is a vast literature on financial crises that we will not review here. See, for example, [Allen and Gale \(2008\)](#), [Boyd et al. \(2005\)](#), and [Thakor \(2012\)](#). On the role of bank capital during crises, [Berger and Bouwman \(2013\)](#) document that higher capital allows banks to capture greater market share during crises.

<sup>2</sup> [Mian and Sufi \(2010, 2011\)](#) have documented a substantial increase in borrower leverage during 2002–06 that was correlated with the increase in house prices. Homeowners extracted 25–30% of the increase in home equity values to increase consumption. They document that the increased borrower leverage during 2002–06 significantly contributed to the higher borrower defaults during 2006–08.

<sup>3</sup> In our model we are analyzing a homebuyer's decision of how much to borrow to buy a *new* house, and how this impacts the borrower's leverage. Given a fixed wealth endowment, the borrower needs to borrow more to buy a more expensive home. This is different from an *existing* homeowner who experiences an increase in home equity due to an appreciation in the price of her home and is not selling it to buy a more expensive house.

borrowers and banks move in unison, is what we call “correlated leverage,” and is our first main result.<sup>4</sup>

An essential element of the analysis is that in addition to the banks’ capital structures, house prices and the borrowers’ capital structures in both periods are also endogenously determined. This rich framework allows us to examine numerous implications of the interaction between the housing market and banks’ leverage choices. We now describe some of these additional results.

The endogeneity of house prices introduces *bank leverage price effects*. Our second main result is that higher first-period bank leverage leads to greater second-period house price volatility. There is thus a transmission from bank capital structure to house prices. The intuition is that more highly-levered banks are less able to absorb negative house price shocks. Banks collect less on their first-period loans in case of a negative fundamental shock to the second-period house price, causing a greater deposit repayment shortfall for more highly-levered banks and introducing credit market fragility. More highly-levered banks need to cover a greater deposit repayment shortfall through fund-raising at a higher marginal cost, and thus supply second-period credit at a higher price, reducing loan demand and further decreasing house prices. Therefore, higher first-period house prices, which lead to higher bank and borrower leverage, result in greater equilibrium house price volatility in the future, making banking and housing more vulnerable to negative shocks.

Our third main result is that a bank’s credit risk exposure is increasing in the leverage choices of *other* banks. That is, bank leverage generates a form of *interconnectedness* among otherwise-independent banks. The intuition is higher leverage of other banks induces higher price volatility and exacerbates the downward price pressure introduced by a negative fundamental shock. This increases the credit risk exposure of even a bank choosing (off the equilibrium path) lower leverage.

Fourth, we extend the model to endogenize banks’ investment in pre-lending screening, and show an increase in house prices leads to “intermediation thinning”, whereby banks invest less in screening borrowers when house prices are higher. This, in contrast to our second main result, highlights a *reverse* transmission mechanism, whereby fundamental shocks to house prices alter financial intermediation and reverberate through the financial sector.<sup>5</sup> The intuition is a higher current house price implies a higher expected future house price, which reduces a bank’s reliance on borrower *income* in collecting loan repayment, thereby diluting the bank’s screening incentive.

We also provide empirical support for some of our results. The first result about correlated leverage and house prices has existing support (over the 2002–08 period), and we discuss this in Section 7. We augment this existing evidence by also documenting that bank leverage and household leverage in the US tended to move in the same direction during 1995–2014. Our second main result that high bank leverage should have a lagged relationship with high house price volatility is a new prediction that we test and find empirical support for. These results are also reported in Section 7.

In addition to these main results, we examine the effect of *ex ante* heterogeneity among banks, with large banks receiving too-big-to-fail (TBTf) protection that small banks do not. There are two main results here. First, TBTf banks diminish the adverse impact of a negative fundamental housing market shock on house prices. The reason is that these banks are insured against the impact of the negative shock on their funding costs, thereby weakening the downward pressure on house prices exerted by the shock. Second, large banks’ leverage choices do not directly affect house prices, but the leverage decisions of small banks do. This is because only small banks (lacking TBTf protection) experience an increase in their funding cost following a negative shock. However, the diminished adverse impact of a negative fundamental shock on house prices due to protected large banks generates an externality that causes *all* banks to increase leverage.

Our analysis generates several policy implications. First, if regulators wish to dampen house price volatility (to reduce individuals’ consumption volatility), they should encourage greater diversification

<sup>4</sup> Although our discussion here considers high first-period house prices, the model is symmetric with respect to the direction of the house price movement. The model predicts that initially-lower house prices are accompanied by lower borrower leverage and lower bank leverage, provided house prices are high enough that homebuyers need bank loans; and second-period house price volatility is lower.

<sup>5</sup> The fundamental shock here is a change in the probability with which a positive shock to the future value of home ownership will occur.

of bank assets. Second, volatility may also be dampened by tying mortgage tax deductions to home ownership duration. Third, bank capital can be linked to *borrower* leverage, with higher bank capital requirements when borrower leverage is higher, or limiting borrower leverage.

The paper proceeds as follows. Section 2 reviews the related literature. Section 3 develops the model. Section 4 contains the preliminaries for the analysis, including a definition of the equilibrium. Section 5 contains the main analysis, and discusses alternative explanations for positively-correlated bank-borrower leverage, along with their relative empirical merits. Section 6 considers extensions. Section 7 summarizes the empirical predictions, provides empirical support for a subset of the predictions, and discusses policy implications. Section 8 concludes. All proofs are in the Appendix.

## 2. Related literature

The purpose of this section is to briefly review various strands of the literature that are related to our work: the literature on bank capital, the literature on household and real estate finance, and the literature that examines the impact of leverage on collateral and credit availability.

There is a vast literature on bank capital, of which the most relevant (e.g., [Inderst and Mueller \(2008\)](#) and [Mehran and Thakor \(2011\)](#)) deals with bank capital structure. A key distinction is that our paper examines not only the capital structure of banks but also how it interacts with the capital structure decisions of borrowers when loans are secured by collateral whose future value is dependent on aggregate bank credit supply. In particular, we show how the endogenous optimizing decisions of banks and borrowers interact to *amplify* the effect of macroeconomic shocks.

The literature on real estate and household finance examines real estate price determination and how it is affected by borrower leverage and various shocks. Examples are [Stein \(1995\)](#), who theoretically examines how down payment restriction affects both prices and trading volume in housing, and [Lamont and Stein \(1999\)](#), who provide empirical evidence for Stein's model. [Kiyotaki and Moore \(1997\)](#) examine the dynamic interaction between credit limits and asset prices. [Fostel and Geanakoplos \(2008\)](#) study how leverage cycles can cause contagion, flight to collateral and issuance rationing in an "anxious economy".<sup>6</sup> [Elul \(2008\)](#) shows how strategic default induced by a drop in the value of collateral in secured borrowing may help stabilize aggregate fluctuations in the housing market. [Khandani et al. \(2013\)](#) show that the interaction between rising home prices, declining interest rates, and increasingly competitive refinancing markets creates a self-synchronizing "ratchet effect," generating systemic risk when home prices fall. Like our work, these papers address the relationship between leverage, credit market frictions and asset prices. The key distinction is that we study the interaction between borrower and bank leverage and its implications for house price dynamics.

The literature on the impact of leverage on collateral and credit availability is the most closely related. In the asset-fire-sales model of [Shleifer and Vishny \(1992\)](#), collateral value depends on other industry peers' ability to buy the asset.<sup>7</sup> When many firms in an industry hold "specialized assets" and all face financial distress due to high leverage, no firm is able to sell to a within-industry firm that will pay a high price for the specialized asset, because all such firms have experienced diminished net worth and borrowing capacity. This forces sales to industry outsiders who value the asset less, which causes a downward price spiral.<sup>8</sup> The similarity is that in our model too there is "common collateral" whose price is affected by an exogenous fundamental shock. Unlike the fire-sale model, however, it is not a loss of industry liquidity – or the constrained purchasing ability of potential buyers – that generates the downward price pressure. Rather, the driving mechanism here is the concomitant endogenous increase in the

<sup>6</sup> There are also papers that examine frictions and the consequent amplification of shocks. See [Cooley et al. \(2004\)](#) and [Kocherlakota \(2000\)](#).

<sup>7</sup> [Benmelech and Bergman's \(2011\)](#) evidence from bankruptcies in the airline industry is consistent with this. See also the literature on "cash-in-the-market" pricing (e.g., [Acharya and Yorulmazer \(2008a\)](#) and [Allen and Carletti \(2008\)](#)). In [Rampini and Viswanathan \(2013\)](#), a firm's leverage is determined by the tangible assets it can use as collateral for borrowing, similar to the role of houses in our model. [Coval and Stafford \(2007\)](#) provide evidence of "fire-sale" induced price pressure in equity markets.

<sup>8</sup> In [Acharya and Viswanathan \(2011\)](#), more highly-levered financial institutions (e.g., broker-dealers) are more likely to face difficulty in rolling over short-term debt upon adverse shocks, and hence de-lever by selling assets to other lower-leverage firms in the sector. Unlike them, we focus on the correlation between borrower and bank leverage and endogenize *both* equity and debt in characterizing bank leverage.

funding costs of banks, which increases the borrowing costs of potential homebuyers and lowers their demand for houses. Moreover, our focus is also different in that we endogenize the capital structures of both banks and borrowers by linking the borrower's future ability to purchase the house to the bank's future ability to lend. Finally, another difference is highlighted by our intermediation thinning analysis, which shows that, in addition to bank leverage, there is a distinct channel through which a higher prior probability of a positive fundamental shock to house prices increases risk, and that is the dilution of the ex ante screening incentives of banks. This screening effect leads to borrower credit quality deteriorating when house prices are rising and collateral-based net worth is high for banks as well as borrowers.

Also related is [Holmstrom and Tirole \(1997\)](#) in which borrowers' access to credit is shown to depend on capital in both banks and borrowers. However, unlike our paper, they take bank and borrower capital as exogenous and do not address why their leverage ratios may be correlated. Moreover, they do not address the impact of these leverage ratios on the housing market.<sup>9</sup>

### 3. The model

#### 3.1. The agents and economic environment

**Time structure:** Consider a three-date ( $t = 1, 2, 3$ ) economy with universal risk neutrality. There are two goods in the economy, money and houses, where money is the numeraire good and can be stored costlessly over time, and houses are indivisible. There is a continuum of atomistic and identical houses available in the market at  $t = 1$  with a measure of  $S$ . There are no new houses built after  $t = 1$ . We call the period between  $t = 1$  and  $t = 2$  the first period, and the period between  $t = 2$  and  $t = 3$  the second period. Discount rates between dates are normalized to zero.

**Consumer preferences:** There is also a continuum of atomistic consumers in each period. Consumers within a given period are identical, but they may differ across periods.<sup>10</sup> A consumer in period  $i \in \{1, 2\}$  is born at  $t = i$  without a house but with a monetary endowment  $M_i > 0$ , and earns an income  $X_i \geq 0$  at  $t = i + 1$ . She maximizes her expected utility at  $t = i$  given by:

$$U_i = h_i B_i + C_i + \mathbb{E}(C_{i+1}) \quad \forall i, \quad (1)$$

where  $h_i$  is an indicator variable that equals 1 if the consumer owns a house in period  $i$  and zero otherwise,  $B_i > 0$  is the consumer's utility from home ownership in period  $i$ , and  $C_i$  and  $C_{i+1}$  are, respectively, the consumer's monetary consumptions at  $t = i$  and  $t = i + 1$ .  $\mathbb{E}(\cdot)$  is the expectation operator.  $B_i$  is meant to capture both the consumption value of the house as well as possible tax benefits from home ownership. The consumption value of the house may be related to the status-related value of conspicuous consumption, utility from "keeping up with the Joneses", or just the innate satisfaction of owning a desirable home stemming from pride of home ownership.

**Value of home ownership and house prices:** In each period the measure of consumers,  $S_c$ , exceeds  $S$ , the housing supply. Each consumer born at date  $t$  takes the house price at  $t$  as given and decides whether to buy a house. First-period consumers who buy houses at  $t = 1$  sell their houses to second-period consumers at  $t = 2$ , who sell theirs to (unmodeled) third-period consumers at  $t = 3$ .<sup>11</sup>

We focus on house prices at  $t = 1$  and 2,  $P_1$  and  $P_2$ , which are endogenously determined based on (among other factors) consumer preferences for home ownership,  $B_1$  and  $B_2$ . While a (first-period) consumer's utility from home ownership in the first period,  $B_1$ , is common knowledge at  $t = 1$ , a

<sup>9</sup> There are also models in which banks become interconnected for reasons other than common collateral. [Farhi and Tirole \(2012\)](#) show that banks' private leverage choices may exhibit strategic complementarities through their reactions to monetary policy. In [Tsomocos et al. \(2007\)](#), under-diversified banks become interconnected through the interbank market. [Acharya and Yorulmazer \(2008b\)](#) show that banks may undertake correlated investments and minimize the impact of information contagion on the expected cost of borrowing.

<sup>10</sup> We allow for heterogeneity among consumers within a given period in an extension of the model in Section 6.2.

<sup>11</sup> The initial housing stock is owned by a generation of consumers that we do not explicitly analyze in the model. These consumers exit the model at  $t = 1$  with either a capital gain or a capital loss, depending on the house price at  $t = 1$ . Alternatively, we could think of the initial housing stock as having been created by builders who financed the construction with their own equity and who exit the model at  $t = 1$  with either a gain or loss on their investment based on the housing price at  $t = 1$ . In either case, the payoffs of house sellers at  $t = 1$  are irrelevant to our analysis.

(second-period) consumer's utility from home ownership in the second period,  $B_2$ , is a random variable at  $t = 1$  that is realized at  $t = 2$ , and this realization influences the resulting house price,  $P_2$ , at  $t = 2$ .<sup>12</sup> Specifically,  $B_2$  takes a high value,  $B_{2h}$ , with probability (w.p.)  $\theta \in (0, 1)$ , and a low value,  $B_{2l}$ , w.p.  $1 - \theta$ , where  $B_{2h} > B_{2l} > 0$ . This preference uncertainty is the only fundamental uncertainty in the housing market, where  $B_{2h}$  represents a positive shock, and  $B_{2l}$  represents a negative shock. A positive (negative) shock increases (decreases) the value that consumers attach to houses and will *ceteris paribus* result in higher (lower) house price. To close the model, we assume that the house price at  $t = 3$ ,  $P_3 \geq 0$ , is exogenously given but random at  $t = 1$  and 2: w.p.  $\beta \in (0, 1)$  it is  $P_{3h} > 0$ , and w.p.  $1 - \beta$  it is  $P_{3l} \in [0, P_{3h})$ . We assume that the fundamental shock at  $t = 2$  and the exogenous shock to house price at  $t = 3$  are independent of each other; they may or may not be identically distributed – we do not impose any relation between  $\theta$  and  $\beta$ .<sup>13</sup>

Note that the preference shock impacts all consumers identically as *all* consumers attach the *same* utility,  $B_i$ , to home ownership in any given period  $i$ . The house prices at  $t = 1$  and 2,  $P_1$  and  $P_2$ , are endogenously determined by competition among the first-period and second-period consumers, respectively, for buying the fixed housing supply,  $S$ .<sup>14</sup> Buying and selling houses involve no transaction costs. It is clear that absent wealth and credit constraints, the house price at  $t = 2$  will be  $P_2 = B_2 + \mathbb{E}(P_3)$ , and the house price at  $t = 1$  will be  $P_1 = B_1 + \mathbb{E}(P_2) = B_1 + \mathbb{E}(B_2 + P_3)$ .

However, consumers are wealth constrained, and their monetary endowments are not large enough to finance home purchases at those prices. Specifically, we assume:

**Assumption 1.**  $M_1 \in (B_1, B_1 + \mathbb{E}(B_2 + P_3))$  and  $M_2 \in (B_2, B_2 + \mathbb{E}(P_3))$ .

This assumption means that consumers' wealth endowments at  $t = 1$  ( $M_1$ ) and  $t = 2$  ( $M_2$ ) are not large enough to completely finance house purchases, so consumers have to borrow. In addition, the assumption,  $M_i > B_i \forall i$ , is made to eliminate the possibility that *all* consumers in period  $i$  *strictly* prefer to purchase a house regardless of the borrowing terms (which occurs if  $M_i \leq B_i$ ); if this possibility is not eliminated, the housing market never clears.

**Consumer borrowing possibilities:** Consumers can borrow from banks by taking mortgage loans, but cannot directly borrow and lend money to each other or get funding from any other source.<sup>15</sup> There is a continuum of atomistic and ex ante identical banks with a measure of  $S/N$ , where  $N$  is a positive constant.<sup>16</sup> In period  $i$ , each bank takes the size of loans demanded by the consumers ( $L_i$ ) and the interest rate on loans ( $R_i$ ) as given, and chooses the number of loans to extend. Bank  $j$  extends  $n_{ij}$  loans in period  $i$ . All loans are for one period: a loan extended at date  $t$  ( $t = 1$  or 2) must be repaid at date  $t + 1$ . No new banks enter at  $t = 2$ .

The consumer can choose whether or not to repay her loan. To minimize the risk of default as well as the loss given default, banks require that each loan be secured by the house purchased using that loan. If a consumer does not repay her loan in full, the bank can seize her house (i.e., foreclose) without any cost and sell it through a foreclosure auction at the prevailing market price of the house.<sup>17</sup> The

<sup>12</sup> There are numerous papers modeling preference shocks like Allen and Gale (2000), Blanchard and Galí (2007), Mace (1991), Ravn et al. (2006), and Weder (2006). Iacoviello (2005) specifies a shock to the marginal rate of substitution between housing and consumption for households to model disturbances that shift housing demand, such as temporary tax advantages to housing investment or a sudden demand increase fueled by optimistic expectations. The preference shock we model may capture these or other factors that impact house prices, such as shocks to endowment of houses or technology of manufacturing houses, or changes in demographics.

<sup>13</sup> There is no connection between  $\theta$  and  $\beta$ , an innocuous assumption that is made merely for analytical simplicity.

<sup>14</sup> We assume that housing supply is inelastic and focus on the interaction between housing demand and credit availability. The assumption facilitates the result that housing prices fall as credit constraints worsen. This result also obtains in Stein (1995). Vigdor (2006) shows that this result holds even when housing supply is elastic. We discuss the robustness of our results with respect to this modeling assumption in Section 5.3.3.

<sup>15</sup> This assumption can be justified on the basis of the specialization of banks as information processors (e.g., Allen (1990) and Ramakrishnan and Thakor (1984)), and specialists in relationship lending (e.g., Boot and Thakor (2000)).

<sup>16</sup> We introduce ex ante heterogeneity among banks in Section 6.1 with too-big-to-fail protection for large banks.

<sup>17</sup> In reality, banks will incur some foreclosure cost and may only get a fraction of the market price of the house in a foreclosure auction. Adding these details into the model does not qualitatively change our results.

bank has no legal claim on the borrower's other assets or income,<sup>18</sup> and similarly the borrower has no legal claim on the bank's proceeds from the sale of the foreclosed house.

### 3.2. Bank capital structure

Banks extend loans using the funds raised through equity and deposits. Each bank independently chooses the amount of equity and deposits. Let  $E_{ij}$  and  $D_{ij}$  be the amount of equity and deposits per loan, respectively, raised by bank  $j$  in period  $i$ . Raising external finance in any form is dissipatively costly, although the source of the cost may vary across equity and debt. The cost of equity may arise due to adverse selection (Myers and Majluf (1984)) and related transactions. Bank  $j$ 's cost of equity in period  $i$  (with the amount of equity  $n_{ij}E_{ij}$ ) is  $\lambda[n_{ij}E_{ij}]^2/2$ , where  $\lambda$  is a positive constant.<sup>19</sup> For deposits, which are assumed to be of the wholesale uninsured variety, this cost may also arise from asymmetric information (balance sheet opaqueness) and transaction costs. The cost is  $\delta[n_{ij}D_{ij}]^2/2$  for bank  $j$  in period  $i$  with the amount of deposits  $n_{ij}D_{ij}$ , where  $\delta$  is a positive constant. The deposit market is perfectly competitive – depositors are promised a competitive expected return of zero. We make no assumption about the magnitude of  $\delta$  relative to  $\lambda$ . The pecking order theory of Myers and Majluf (1984) would suggest  $\delta < \lambda$ . While our analysis accommodates such a relationship, our results do not depend on how  $\delta$  and  $\lambda$  compare with each other.

The quadratic functional forms for the cost functions are assumed merely for analytical tractability in that they help to generate interior solutions. Papers such as Mehran and Thakor (2011) have made a similar assumption. They also help to capture the idea that is commonly put forth by bankers that their marginal cost of funding from any source goes up as they tap that source to a greater extent. Moreover, we assume the same functional forms for the cost of equity and the cost of deposits to ensure that our main results on bank leverage are driven by the economic forces identified in the model, and not by any asymmetry in the cost functions for debt and equity.

A bank may not be able to fully repay depositors from its first-period loan proceeds at  $t = 2$ . Suppose the repayment shortfall is  $z$ . In that case the bank needs to raise funds to not only finance its second-period loans, but also to cover  $z$  from the previous period.<sup>20</sup> These additional funds, as our later analysis shows, increase the bank's second-period cost of financing and lending and hence represent a cost of leverage that affects the bank's first-period capital structure. The parametric assumption below ensures that banks may experience a deposit repayment shortfall in our model:

**Assumption 2.** The negative shock to the housing market is sufficiently strong, i.e.,  $B_{2l} < \hat{B}$ , where the upper bound  $\hat{B}$  is defined in the Appendix.

This assumption ensures that the equilibrium second-period house price, conditional on a negative fundamental shock ( $B_2 = B_{2l}$ ), is sufficiently low so that first-period borrowers default on their loans and houses are foreclosed, yet banks' proceeds from the sale of the foreclosed houses are insufficient to repay their depositors in full. The Appendix shows that, in equilibrium, the first-period homeowners repay their loans, conditional on a positive shock ( $B_2 = B_{2h}$ ).

<sup>18</sup> We relax this no-recourse assumption in Section 6.2, where we examine the bank's intermediation role and model a loan with (limited) recourse that allows the bank to claim part of the borrower's income upon default on her loan.

<sup>19</sup> We assume that regulatory capital requirements, if they exist, are not binding. Binding regulatory capital requirements will dampen the time-series variation in bank capital ratios and weaken the correlated leverage results. Note, however, that there is no endogenous rationale for capital requirements in our model. The assumption that bank equity capital is costly is fairly standard in banking models. See, for example, Allen et al. (2011) and Mehran and Thakor (2011). The exogenous specification of the dissipative cost makes it general enough to incorporate a variety of sources that could contribute to the cost. Our cost function for deposits has a functional form that is similar to that for equity.

<sup>20</sup> The bank is assumed to experience sufficiently high dissipative costs (e.g., loss of charter value) if it actually defaults on its deposits, so an uninsured bank is always willing to incur the additional second-period cost to meet its first-period deposit repayment shortfall. Thus, there are no bank failures in the model. This no-failure assumption does not affect our results qualitatively; indeed, some of our results are strengthened if we allow bank failures.

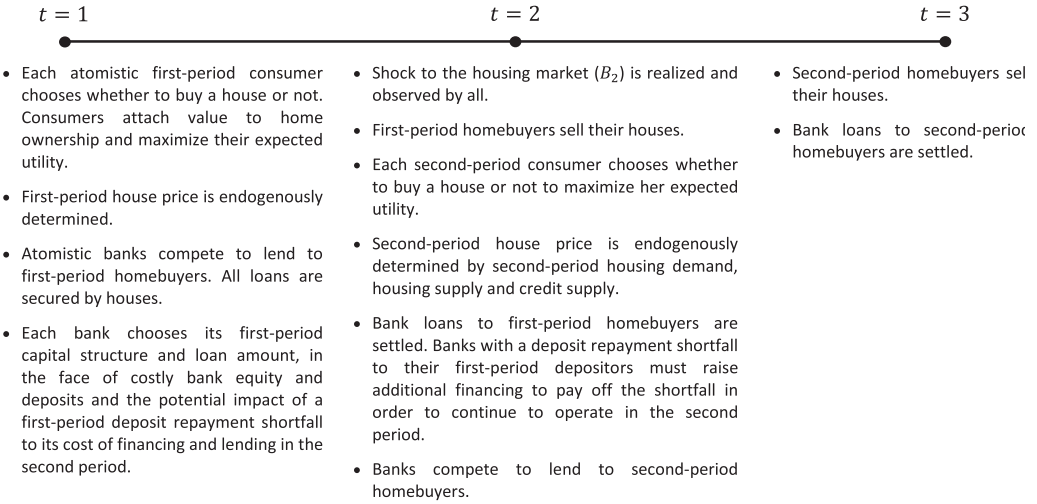


Fig. 1. Sequence of events.

Each bank maximizes its expected second-period profit at  $t = 2$  by choosing the number of loans to extend and the second-period capital structure. At  $t = 1$ , each bank chooses the number of loans and the first-period capital structure to maximize the sum of its expected profits in both periods, accounting for the effect of a deposit repayment shortfall in the first period on its second-period financing. In each period, banks take loan size and loan interest rate as given (which are determined in a competitive equilibrium – see Section 4.3), since each bank is atomistic. The size of each loan is determined by the amount that a homebuyer needs to borrow. A bank can determine the size of its own loan portfolio by choosing the number of loans (possibly a non-integer) it makes. If a borrower’s loan is financed by multiple banks, a bank’s fraction of the loan repayment equals the fraction of the borrower’s loan that the bank provided. Fig. 1 summarizes the events sequence.

**4. The analysis: some preliminaries**

This section presents preliminaries for an analysis of the model. We first examine trading in the housing market, describe the bank’s problems in the two periods, analyze the determination of the bank’s capital structure and equilibrium in the loan market in terms of the number of loans the bank chooses to make. We subsequently define the overall equilibrium.

*4.1. Housing market and the determination of equilibrium house prices*

House prices in period  $i \in \{1, 2\}$  are determined by consumers competing for a fixed housing supply:

$$\text{House Supply}_i = S \quad \forall i. \tag{2}$$

A consumer purchasing a house at  $t = i$  uses her endowment  $M_i$  as down payment and takes a bank loan with the size:<sup>21</sup>

$$L_i = P_i - M_i \quad \forall i. \tag{3}$$

Let  $R_i > 1$  be the gross interest rate charged by the bank in period  $i$ , which will be endogenously determined later. This rate is independent of the consumer or bank identity because of our assumption of identical consumers and banks. A consumer’s expected utility from buying a house at  $t = i$  is obtained

<sup>21</sup> Note that the loan size,  $L_i$ , cannot be negative in equilibrium because Assumption 1 rules out  $P_i \leq M_i$ .



from (1) and consists of three parts. The first is the utility from home ownership in period  $i$  ( $B_i$ ) obtained by substituting  $h_i = 1$  in (1). Since the homeowner uses his endowment as down payment, his consumption at the beginning of the period is  $C_i = 0$ . Finally, the consumption at the end of the period,  $C_{i+1}$ , consists of her income at  $t = i + 1$  ( $X_i$ ) and the expected gain from house price appreciation in period  $i$  after paying off the bank loan (which is the maximum of zero and the excess of the house price at  $t = i + 1$ ,  $P_{i+1}$ , over the repayment to the bank,  $R_i L_i$ ):

$$U_i^b = B_i + X_i + E((P_{i+1} - R_i L_i)^+) \quad \forall i, \tag{4}$$

where  $\omega^+$  denotes the maximum of  $\omega$  and 0. The expected utility of a consumer who does not buy a house is obtained from (1) by setting  $h_i = 0$ , start-of-period consumption  $C_i$  equal to her endowment ( $M_i$ ), and end-of-period consumption  $C_{i+1}$  equal to her income ( $X_i$ ):

$$U_i^{nb} = M_i + X_i \quad \forall i. \tag{5}$$

Thus, the demand of houses at  $t = i$  is:

$$\text{House Demand}_i \begin{cases} = 0 & \text{if } U_i^b < U_i^{nb}, \\ \in [0, S_c] & \text{if } U_i^b = U_i^{nb}, \\ = S_c & \text{if } U_i^b > U_i^{nb}, \end{cases} \quad \forall i, \tag{6}$$

where  $S_c$  is the measure of consumers in each period, exceeding housing supply in that period,  $S$ .

In equilibrium,  $\text{House Demand}_i = \text{House Supply}_i$ , so consumers are indifferent between purchasing a house and not purchasing one:

$$M_i = B_i + E((P_{i+1} - R_i L_i)^+) \quad \forall i. \tag{7}$$

The market-clearing condition (7) determines the equilibrium house price  $P_i$ , taking the interest rate  $R_i$  as given.<sup>22</sup> The left-hand side of (7) is the monetary consumption that the consumer gives up in buying a house, and its right-hand side consists of the utility from home ownership and the expected gain from house price appreciation after paying off the loan.

#### 4.2. The bank's optimization problems in the first and second periods

In this subsection, we describe the bank's optimization problems in the first and second periods, as a prelude to examining the bank's optimal capital structure in the first period.<sup>23</sup>

##### 4.2.1. The second period

Consider bank  $j$ 's problem in the second period when the realized shock is  $B_2 = B_{2k}$ ,  $k \in \{h, l\}$ . The bank's first-period deposit repayment shortfall is  $n_{1j}(D_{1j} - P_{2k})^+$ , where  $P_{2k}$  is the equilibrium second-period house price conditional on  $B_2 = B_{2k}$ . Thus, in addition to the funds needed to finance its second-period loans, bank  $j$  also needs to raise  $n_{1j}(D_{1j} - P_{2k})^+$  to cover that shortfall. That is,

$$n_{2jk}[D_{2jk} + E_{2jk}] = n_{2jk}L_{2k} + n_{1j}(D_{1j} - P_{2k})^+ \quad \forall j, k, \tag{8}$$

where  $L_{2k}$ ,  $n_{2jk}$ ,  $D_{2jk}$  and  $E_{2jk}$  are the second-period loan size, the number of loans extended by bank  $j$ , deposits and equity per loan by bank  $j$ , respectively, conditional on  $B_2 = B_{2k}$ .

**Bank's second-period profit:** Without loss of generality, we assume  $P_{3h}$  to be sufficiently large to cover the contractually-stipulated repayment for the second-period loan, and normalize  $P_{3l}$  to zero.<sup>24</sup> Thus, bank  $j$ 's expected second-period profit, conditional on  $B_2 = B_{2k}$ , is given by:

$$\pi_{2jk} = n_{2jk}[\beta R_{2k}L_{2k} - L_{2k}] - \frac{\delta[n_{2jk}D_{2jk}]^2}{2} - \frac{\lambda[n_{2jk}E_{2jk}]^2}{2} \quad \forall j, k, \tag{9}$$

<sup>22</sup> The competition in the housing market uniquely determines the loan repayment amount  $R_i L_i$ , but not the house price  $P_i = L_i + M_i$ , which also depends on the interest rate  $R_i$  charged by the banks.

<sup>23</sup> We also analyze bank capital structure in the second period, but our main result focuses on the first period.

<sup>24</sup> Allowing  $P_{3l} > 0$  does not change anything qualitatively, since the profitability of the second-period loan for the bank depends only on the expected value of the house price at  $t = 3$ .

where  $R_{2k}$  is the second-period loan interest rate conditional on  $B_2 = B_{2k}$ . There are three terms in  $\pi_{2jk}$ . The first term equals the number of loans the bank chooses to make,  $n_{2jk}$ , multiplied by the expected repayment from the borrower per loan,  $\beta R_{2k} L_{2k}$ , net of the loan extended,  $L_{2k}$ . The last two terms are deductions for the dissipative costs of deposit gathering and equity capital.<sup>25</sup>

Each (atomistic) bank  $j$  takes the loan size ( $L_{2k}$ ) (and hence the house price  $P_{2k}$ ) and the loan interest rate ( $R_{2k}$ ) as given, and chooses its second-period capital structure ( $D_{2jk}$  and  $E_{2jk}$ ) and the loan amount ( $n_{2jk}$ ) to maximize  $\pi_{2jk}$ .

**Bank's optimal second-period capital structure:** The first-order condition for a capital structure that maximizes  $\pi_{2jk}$  is:

$$\frac{D_{2jk}}{E_{2jk}} = \frac{\lambda}{\delta} \quad \forall j, k. \tag{10}$$

**Loan market equilibrium:** The aggregate second-period loan demand is the measure of consumers who purchase houses, i.e.,  $Loan\ Demand_{2k} = House\ Demand_{2k}$ . Consumers' housing demand depends both on the loan interest rate and the house price. When the housing market is in equilibrium, competition among consumers ensures that the equilibrium house price adjusts so that the measure of consumers who purchase houses equals the supply of houses,  $S$ . Thus,  $Loan\ Demand_{2k} = S$  when the housing market clears in equilibrium. The aggregate loan supply is:

$$Loan\ Supply_{2k} = \int_{S/N} n_{2jk} dj \quad \forall k. \tag{11}$$

Bank  $j$ 's first-order condition for a choice of  $n_{2jk}$  that maximizes  $\pi_{2jk}$  in (9) is:

$$\beta R_{2k} L_{2k} - L_{2k} = \lambda n_{2jk} E_{2jk}^2 + \delta n_{2jk} D_{2jk}^2 = \lambda E_{2jk} n_{2jk} [D_{2jk} + E_{2jk}] = \lambda E_{2jk} [n_{2jk} L_{2k} + n_{1j} (D_{1j} - P_{2k})^+] \quad \forall j, k, \tag{12}$$

where the second equality follows from (10) and the third equality uses (8). The left-hand side (LHS) of (12) is the expected net return on the marginal loan, and the right-hand side (RHS) is the marginal cost of lending for the bank. The convex costs of equity and debt result in a marginal cost of lending for the bank that is increasing in the amount of the first-period deposit repayment shortfall,  $n_{1j} (D_{1j} - P_{2k})^+$ , which is the additional financing that the bank has to raise in the second period beyond what is required to finance the loans ( $n_{2jk} L_{2k}$ ). As will be made clear later in our analysis, this feature of the model allows us to examine how bank leverage in the first period affects the bank's marginal cost of lending in the second period and hence the second-period house price.

Note from (12) that an increase in the second-period loan interest rate,  $R_{2k}$ , raises the bank's marginal return from each loan above its marginal cost (LHS > RHS) and causes the bank to lend more ( $n_{2jk}$  increases) until the marginal return and marginal cost become equal (LHS = RHS). Thus, each bank's supply of loans is increasing in  $R_{2k}$ , and so is the aggregate loan supply in (11).

Equilibrium in the loan market requires that the loan interest rate is such that  $Loan\ Demand_{2k} = Loan\ Supply_{2k}$ . The reason for this is the standard market-clearing argument for equilibrium.<sup>26</sup> Equating the loan demand  $S$  to the supply in (11), in a symmetric equilibrium we must have:

$$n_{2jk} = N \quad \forall j, k. \tag{13}$$

<sup>25</sup> The fund raised to resolve the bank's first-period deposit repayment shortfall,  $n_{1j} (D_{1j} - P_{2k})^+$ , does not enter the bank's continuation utility in the second period. This is because the shortfall is what the bank owes to its depositors in the first period, and it has been reflected in the bank's first-period profit calculation. This will be made clear later in (14) in Section 4.2.2; see footnote 27 for details.

<sup>26</sup> Consider a loan rate  $R_{2k}$  such that  $Loan\ Supply_{2k} > Loan\ Demand_{2k}$ . This means some banks extend less than the profit-maximizing number of loans. This cannot be an equilibrium because these banks can increase profits by charging a rate lower than  $R_{2k}$ , increasing demand for their loans. Suppose  $R_{2k}$  is such that  $Loan\ Supply_{2k} < Loan\ Demand_{2k}$ , in which case some consumers cannot get loans. This cannot be an equilibrium either because a bank can increase its profit by extending more loans at a rate higher than  $R_{2k}$ . Our definition of equilibrium includes incentive-compatibility conditions for banks' choices of loan amount and capital structure, but not for  $R_{2k}$  which is determined by the market-clearing condition above. Nonetheless, no bank wishes to deviate from  $R_{2k}$  at which loan demand and supply are equated. If a bank charges more than  $R_{2k}$ , there will be no demand for its loans and its profit will be zero. If it charges less than  $R_{2k}$ , the marginal revenue from each loan will decline, so the bank will not only optimally reduce the loan amount but also earn lower profits on the remaining loans.

That is, the number of loans made by each bank in the second period equals  $N$ , the ratio of the measure of houses ( $S$ ) to the measure of banks ( $S/N$ ).

4.2.2. The first period

**Bank's first-period profit:** Bank  $j$ 's first-period expected profit is:

$$\pi_{1j} = n_{1j}[\theta R_1 L_1 - L_1] - \frac{\delta[n_{1j}D_{1j}]^2}{2} - \frac{\lambda[n_{1j}E_{1j}]^2}{2} \quad \forall j, \tag{14}$$

whose interpretation is similar to that of  $\pi_{2jk}$  in (9). In deriving (14), we have used Assumptions 1 and 2, which ensure that: (i) a first-period borrower only repays her loan in full ( $R_1 L_1$ ) when the housing market experiences a positive shock (i.e.,  $B_2 = B_{2h}$ ), which occurs w.p.  $\theta$ ; and (ii) when the housing market experiences a negative shock (i.e.,  $B_2 = B_{2l}$ ), which occurs w.p.  $1 - \theta$ , the bank forecloses the borrowers' houses, but the proceeds from the sale of the foreclosed houses are insufficient to repay the depositors in full, so nothing is left for the bank in this state.<sup>27</sup>

**Bank's expected profits across two periods:** Since both equity capital and deposits are costly and banks do not have any other investment opportunities, no bank will raise more funds than needed to finance its loan in the first period. That is,

$$D_{1j} + E_{1j} = L_1 \quad \forall j. \tag{15}$$

At  $t = 1$  the bank chooses its first-period capital structure ( $D_{1j}$  and  $E_{1j}$ ) and the amount of loans ( $n_{1j}$ ) to maximize its total expected profits across two periods ( $\pi_{2jh}$  and  $\pi_{2jl}$  are given by (9)):

$$\pi_{1j} + \theta\pi_{2jh} + [1 - \theta]\pi_{2jl} \quad \forall j. \tag{16}$$

**Bank's first-period optimal capital structure:** We now examine the bank's first-period capital structure choice to maximize (16). Assumptions 1 and 2 ensure that there is no bank-deposit-repayment shortfall conditional on a positive shock to the housing market. Thus,  $n_{1j}(D_{1j} - P_{2h})^+ = 0$  (i.e., no deposit repayment shortfall in this state) and hence  $\partial\pi_{2jh}/\partial D_{1j} = 0$ . However, Assumption 2 also ensures that, conditional on a negative shock to the housing market, the bank will experience a first-period deposit repayment shortfall and hence has to raise additional financing in the second period. It can be shown that (see the Appendix for details):

$$\frac{\partial\pi_{2jl}}{\partial D_{1j}} = -n_{1j}n_{2jl} \left[ \frac{\lambda\delta}{\lambda + \delta} \right] \left[ L_{2l} + \frac{n_{1j}}{n_{2jl}} [D_{1j} - P_{2l}] \right] < 0 \quad \forall j. \tag{17}$$

To understand why the bank's second-period profit, conditional on a negative shock, is decreasing in its first-period deposits, note that a larger deposit volume in the first period results in a bigger deposit repayment shortfall conditional on a negative housing market shock. This increases the financing that the bank must raise in the second period, which elevates its marginal financing cost and decreases its second-period profit. Thus, when each bank chooses its first-period capital structure, it must internalize the adverse impact of the first-period deposits on its second-period profit conditional on a negative housing market shock. Note that an increase in a bank's first-period leverage does not impact the second-period house price or interest rate since each bank is atomistic.

The first-order condition for a profit-maximizing first-period capital structure is thus:

$$\lambda E_{1j} = \delta D_{1j} + [1 - \theta] \left[ \frac{n_{2jl}}{n_{1j}} \right] \left[ \frac{\lambda\delta}{\lambda + \delta} \right] \left[ L_{2l} + \frac{n_{1j}}{n_{2jl}} [D_{1j} - P_{2l}] \right] \quad \forall j. \tag{18}$$

**Number of first-period loans made by bank:** Next, we analyze the bank's choice of the number of first-period loans,  $n_{1j}$ . Again, from Assumptions 1 and 2 we have  $\partial\pi_{2jh}/\partial n_{1j} = 0$ . It can be shown that (see the Appendix for details):

<sup>27</sup> Note that although the bank cannot repay its depositors in full in this state with a negative shock, we still deduct the entire loan,  $n_{1j}L_1$ , from the calculation of  $\pi_{1j}$ , since the deposit repayment shortfall will be paid by the bank in the second period (at  $t = 2$ ) via additional fund raising. This also clarifies why the additional fund raised to resolve the shortfall does not enter the bank's second-period continuation utility calculation in (9).

$$\frac{\partial \pi_{2jl}}{\partial n_{1j}} = -n_{2jl} \left[ \frac{\lambda \delta}{\lambda + \delta} \right] \left[ L_{2l} + \frac{n_{1j}}{n_{2jl}} [D_{1j} - P_{2l}] \right] [D_{1j} - P_{2l}] < 0 \quad \forall j. \tag{19}$$

The reason why the bank’s second-period profit, conditional on a negative shock, is decreasing in the number of loans it extends in the first period, is that bank  $j$ ’s deposit repayment shortfall conditional on a negative shock is *ceteris paribus* larger when the bank extends more first-period loans (larger  $n_{1j}$ ). Here again, when each (atomistic) bank internalizes the adverse impact of its first-period loan amount on its second-period profit, it takes the second-period house price as given.

Thus, the first-order condition for a profit-maximizing first-period loan amount is:

$$\theta R_1 L_1 - L_1 = \lambda n_{1j} E_{1j}^2 + \delta n_{1j} D_{1j}^2 + [1 - \theta] n_{2jl} \left[ \frac{\lambda \delta}{\lambda + \delta} \right] \left[ L_{2l} + \frac{n_{1j}}{n_{2jl}} [D_{1j} - P_{2l}] \right] [D_{1j} - P_{2l}] \quad \forall j. \tag{20}$$

Finally, following the same argument in Section 4.2.1, in a symmetric equilibrium we have:

$$n_{1j} = N \quad \forall j. \tag{21}$$

That is, the equilibrium number of loans made by each bank in the first period also equals  $N$ .

### 4.3. The equilibrium in the housing and loan markets

We now define a symmetric equilibrium (across ex ante identical banks) involving house prices, loan sizes, loan interest rates, consumer leverage and bank capital structure choices in both periods.

**Definition of Equilibrium:** A competitive, subgame perfect, rational expectations equilibrium consists of the house prices  $(P_1, P_{2h}, P_{2l})$ , loan sizes  $(L_1, L_{2h}, L_{2l})$ , loan interest rates  $(R_1, R_{2h}, R_{2l})$  in each state (first period or second period with realization of  $B_{2h}$  or  $B_{2l}$ ), and the amount of bank deposits and equity capital per loan in the first period  $(D_{1j}$  and  $E_{1j})$  and second period  $(D_{2jk}$  and  $E_{2jk}$ , conditional on realization of  $B_{2k}$ , where  $k \in \{h, l\}$ ), such that:

1. Each consumer chooses whether to buy a house (expected utility in (4)) or not (expected utility in (5)) to maximize her expected utility, taking the house price and the loan interest rate as given. The consumer’s choice in any period determines her leverage in that period, given the loan size and the consumer’s monetary endowment. The loan size is just sufficient to allow a consumer to buy a house given her endowment (see (3)).
2. Each bank, indexed  $j$ , chooses its loan amount  $(n_{2jk})$  and capital structure  $(D_{2jk}$  and  $E_{2jk})$  in the second period, subject to (8), to maximize its expected profit in that period (9), conditional on realization of  $B_{2k}$ , where  $k \in \{h, l\}$ , taking the loan interest rate  $(R_{2k})$  and the loan size  $(L_{2k})$  as given. In the first period, each bank  $j$  chooses the number of first-period loans  $(n_{1j})$  to make and its capital structure  $(D_{1j}$  and  $E_{1j})$ , subject to (15), to maximize the total expected profits from both periods (16).
3. In each period, banks and consumers form (rational) expectations about the future house price and the actions of other banks and consumers that are consistent with the equilibrium actions of banks and consumers and the expected future house price.
4. House demand equals house supply, and loan demand equals loan supply.

The equilibrium house prices, loan interest rates, and banks’ equity and deposits per loan are jointly determined by the consumers’ indifference condition (7), the banks’ optimal capital structure conditions (10) and (18), the banks’ optimal loan amount conditions (12) and (20), and the loan market clearing conditions (13) and (21).

## 5. Analysis of the equilibrium in the loan and housing markets

This section analyzes the previously defined equilibrium and derives three main results. In the usual dynamic programming manner, we use backward induction beginning with the second period.

### 5.1. Equilibrium in the second-period subgame

The main variables of interest are the second-period equilibrium house price and its impact on the credit risk exposures of banks with respect to their first-period loans. Our first result deals with the relationship between the equilibrium house price and the value of the fundamental shock,  $B_2$ .

#### 5.1.1. Shock to fundamentals and the second-period house price

**Lemma 1.** *Given the first-period outcomes, the subgame in the second period has a unique equilibrium in which the house price at  $t = 2$ ,  $P_2$ , is increasing in a second-period consumer's utility from home ownership in the second period,  $B_2$ .*

As the second-period consumer's utility from home ownership,  $B_2$ , increases, houses become more attractive if house price and loan rate do not change. Housing demand exceeds (fixed) housing supply. Equilibrium is restored with an increase in the second-period house price and loan rate.

#### 5.1.2. Bank leverage and the second-period house price

Next, we examine how the first-period leverage choices of banks affect the second-period house price. The following result is a comparative statics analysis of the equilibrium about the consequence of higher bank leverage in the first period for the second-period house price.

**Proposition 1.** *If all banks choose higher leverage at  $t = 1$  as part of the overall equilibrium over the two periods, then conditional on a negative fundamental shock ( $B_2 = B_{2l}$ ), the second-period house price,  $P_{2l}$ , experiences a greater decline in response to the shock. However,  $P_{2h}$ , the house price conditional on a positive shock ( $B_2 = B_{2h}$ ), is unaffected by bank leverage chosen at  $t = 1$ . Thus, a variation in exogenous parameters that leads to a higher first-period bank leverage in equilibrium leads to higher volatility in the equilibrium second-period house price.*

Banks' first-period leverage choices at  $t = 1$  are based on the probability distribution of  $B_2$ . In describing the overall two-period equilibrium in Proposition 3, we show how this endogenous choice of first-period bank leverage is linked to an exogenous (deep) parameter,  $\theta$ . Proposition 1 deals with how, conditional on this earlier leverage choice, in the second-period subgame the equilibrium house price at  $t = 2$  responds to the realization of the fundamental shock,  $B_2$ .

The intuition is that the higher the banks' first-period leverage, the higher is the volatility of their first-period deposit repayment shortfalls, leading to higher volatility of the banks' second-period loan terms. These loan terms, in turn, affect second-period housing demand and price.

To see this in greater detail, note that higher first-period leverage results in larger bank deposit repayment shortfalls when the housing market suffers a negative fundamental shock ( $B_2 = B_{2l}$ ) at  $t = 2$ . That is, higher bank leverage amplifies the adverse impact of the negative fundamental shock in the housing market by increasing the amount of bank deposit repayment shortfalls. As a result, banks need to raise more funds in the second period. This causes their marginal cost of lending in the second period to increase, leading to a higher second-period loan interest rate. But this lowers the second-period consumers' demand for loans to finance house purchases, causing the equilibrium house price  $P_{2l}$  at  $t = 2$  to decline further in order to clear the housing market. We have thus established the second of the four main results mentioned in the Introduction – higher bank leverage in the first period causes higher volatility in the second-period house price.

Note that  $P_{2h}$  is independent of the first-period leverage choices of banks. This is because a bank's first-period profit, conditional on a positive shock to the housing market ( $B_2 = B_{2h}$ ), does not affect its second-period financing and hence cost of lending. An implicit assumption here is that banks pay out all their earnings in this state as dividends. What if banks retained their earnings in order to rely less on outside financing in the second period and thereby lower their cost of funds? If we were to make this assumption, then to the extent that raising external capital is dissipatively costly, higher first-period leverage would increase the banks' first-period realized profits in the state when  $B_2 = B_{2h}$ , which further lowers the banks' cost of funds in the second period and, in turn, leads to a higher

$P_{2h}$ . This will result in an even larger impact of leverage on house price volatility than in our analysis. That is, such a change in the model will only make our result stronger.

Another assumption in the analysis is that there are no bank failures. If bank failures were allowed, our result that higher first-period bank leverage leads to higher second-period house price volatility would be strengthened. To see this, suppose some banks, for reasons outside the model, choose not to raise additional funds at  $t = 2$  (or are unable to do so) to repay their first-period deposit shortfalls when  $B_2 = B_{2j}$  and hence fail. Then, there will be fewer banks financing home purchases in the second period, and hence *ceteris paribus* each surviving bank will need to raise more funds to meet the demand for loans, which will drive up the marginal cost of bank loans and lower the equilibrium house price  $P_{2i}$  even further as compared to the case without bank failures.

### 5.1.3. The effect of the leverage choices of other banks on a bank's credit risk exposure

Next, we consider an extension of Proposition 1. Bank  $j$ 's expected deposit repayment shortfall at  $t = 2$ , defined as its "credit exposure," is:

$$\mathbb{E}(n_{1j}(D_{1j} - P_2)^+). \quad (22)$$

Note bank  $j$ 's credit exposure depends on  $P_2$ , which, in turn, depends on other banks' first-period leverage choices (Proposition 1). The fact that all the banks' loans are backed by the same collateral (i.e., houses) engenders interconnectedness among otherwise-independent banks. We thus have:

**Proposition 2.** *Given the first-period outcomes, in the second-period subgame each bank's credit risk exposure at  $t = 2$  is increasing in the first-period leverage ratios of all other banks.*

This result shows how leverage contributes to bank interconnectedness. For each bank, higher first-period leverage of other banks means a greater second-period house price decline in response to a negative shock, implying lower collateral values at all banks, and higher credit risk exposure for each bank. Interconnectedness is thus generated by the impact of bank leverage on the value of common collateral. This establishes the third of the four main results discussed in the Introduction.

## 5.2. Overall equilibrium in the first period

The preceding analysis took as given the first-period choice variables. We now study the endogenous determination of these variables as part of the overall (two-period) equilibrium and hence the relationship between house prices, consumer leverage, and bank leverage in the first period.

**Proposition 3.** *There exists a unique subgame perfect equilibrium for the two periods. The subgame defined by the second period involves the equilibrium that was described in Proposition 1. In the first period, all banks choose the same equilibrium capital structure (deposits  $D_1$  and equity  $E_1$ ). First-period consumer leverage  $L_1/M_1$ , bank leverage  $D_1/E_1$ , and house price  $P_1$  are all increasing in  $\theta$ , the probability of a positive fundamental shock to the value attached to home ownership by second-period homebuyers.*

This is the correlated leverage result, the first main result discussed in the Introduction. The intuition is as follows. Housing demand depends on a comparison that a consumer makes between the benefits of home ownership – the utility associated with home ownership and the expected house price appreciation during the period of ownership – with the price she pays for the house. Consider an increase in  $\theta$ , the probability of a high value of second-period home ownership. *Ceteris paribus* it makes house price appreciation more likely, causing aggregate housing demand to increase at  $t = 1$ . A market-clearing equilibrium is restored when housing demand is lowered via two channels: banks increase the loan interest rate in response to the increased loan demand, and consumers compete more aggressively with each other to buy houses and bid up the price. The higher first-period house price causes borrowers, who have fixed initial wealth endowments, to ask for bigger bank loans, leading to higher borrower leverage. Moreover, since an increase in  $\theta$  diminishes the probability  $(1 - \theta)$  of a decline in the house price at  $t = 2$ , banks' credit risk declines because borrowers' loan repayments

are predicated on the future value of the house as collateral. This reduces the marginal benefit of equity capital to banks as a cushion to absorb credit risk, so banks keep lower capital precisely when borrowers are more highly leveraged, generating correlated leverage.

Taken together, [Propositions 1 and 3](#) imply that as a positive fundamental price shock becomes more likely, both borrowers and banks become more highly levered, and second-period house price volatility spikes up. This helps to understand the comparative static result in [Proposition 1](#) in terms of a deep parameter of the model ( $\theta$ ), rather than an endogenous choice variable.

### 5.3. Robustness

#### 5.3.1. Alternative explanations for correlated leverage and their empirical merits

One alternative explanation is that a decline in interest rates may make borrowing more attractive for both banks and consumers, thereby driving up both bank and consumer leverage. While intuitively plausible, this does not seem to be what was going on prior to the subprime crisis. Quarterly data during the 1990s reveal that bank capital ratios rose gradually even after exceeding regulatory capital requirements ([Flannery and Rangan, 2008](#)) during a period of falling interest rates, i.e., bank leverage rose as interest rates declined. Besides lacking empirical support, the theoretical foundation of this explanation is also shaky – it relies on the assumption that lower interest rates do not lower the cost of bank equity by as much as they lower the cost of bank debt.

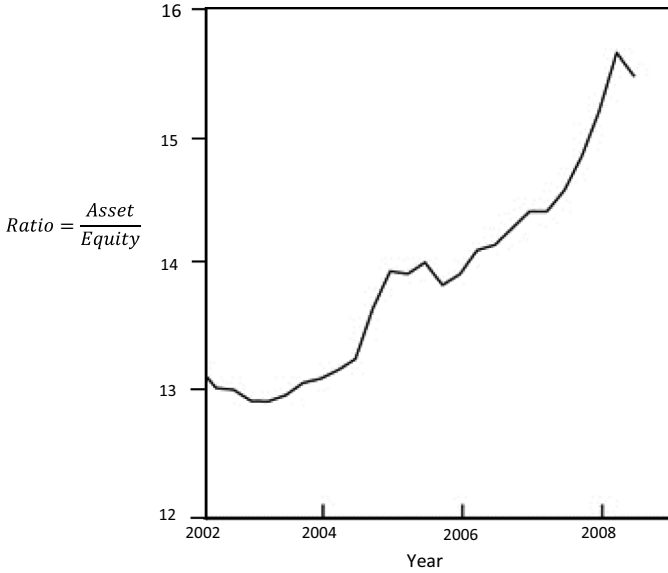
Another possible explanation comes from lending technology shocks. Improvements in lending technologies that lower banks' funding costs – securitization is an example – or improve risk sharing, with some benefits going to consumers, can encourage consumer borrowing, thereby increasing consumer leverage. If the reduction in the funding costs of banks is greater at the short end of the maturity spectrum than at the long end, then short-maturity bank leverage would also increase. While plausible, this story does not help us understand why this leverage correlation arises when real estate prices are booming (see [Fig. 2](#) in [Section 7.2](#)).

#### 5.3.2. Heterogenous houses

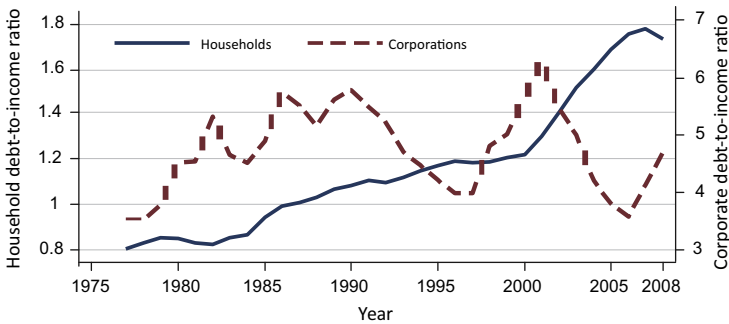
We have assumed that houses are identical in size. This immediately implies that borrowers need to borrow more when house prices go up. With heterogeneity in house size, a borrower could avoid borrowing more by opting for a smaller and cheaper house. How would that affect our results? To examine this, consider an extension of the model in which houses differ in their sizes and consumers *ceteris paribus* prefer bigger houses. To fix ideas, suppose houses are of two sizes: big and small. A consumer facing an increase in the price of big houses may choose to buy a less expensive small house. However, this will not necessarily decrease consumer leverage. The reason is that the supply of houses is fixed for each size cohort, so not all consumers can move from a big house to a small house. The number of consumers who can buy houses of a given size in equilibrium equals the housing supply for that size cohort, and the equilibrium prices will be such that consumers are indifferent between buying big and small houses. A demand shift from big houses to small houses, possibly due to an increase in the price of big houses, will increase the demand for small houses, which consequently increases the price of small houses as well, again driving up consumer leverage.

#### 5.3.3. Endogenous housing supply

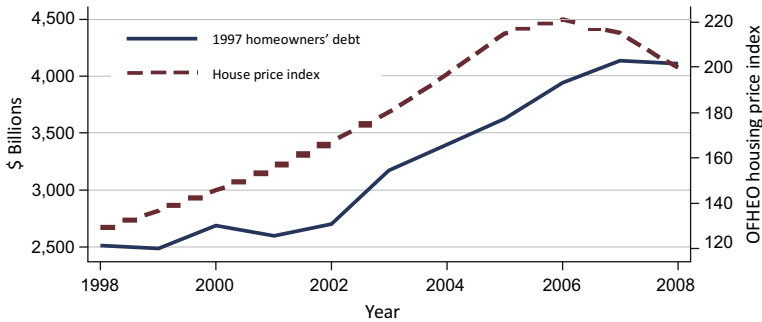
Our model is also robust to endogenizing housing supply. One can view the model as focusing on a “short-run” equilibrium with a fixed housing stock, but our main results extend to a “long-run” equilibrium with elastic housing supply. The idea is as follows. Suppose  $\theta$  increases to  $\theta'$ , so at the old equilibrium price  $P_1$ , demand for houses increases. There are two ways to establish a new equilibrium: (i) keep the housing supply fixed but increase its price in response to the additional demand (as in our “short-run” equilibrium), or (ii) allow supply to be increasing in price, so that supply can increase to satisfy the additional demand. The key is that even in the latter case, the new equilibrium price, call it  $P'_1$ , has to be higher than  $P_1$ : if  $P'_1 \leq P_1$ , then housing supply will not increase in the first place. Of course, the magnitude of the house price increase in response to a higher  $\theta$  is smaller when housing supply is elastic, but our results sustain qualitatively.



(a) U.S. bank (commercial and investment) leverage



(b) U.S. debt-to-income ratio: households and corporations



(c) U.S. household debt for 1997 homeowners and house prices

**Fig. 2.** Aggregate US bank, household and corporate leverage and house price. (a), adapted from [Hatzius \(2008\)](#), plots the aggregate leverage ratios of US banks (commercial and investment) since 2002, where leverage is defined as total assets divided by equity capital. (b) and (c), adapted from [Mian and Sufi \(2011\)](#), show the aggregate US household and corporate leverage ratios and house price patterns.



#### 5.3.4. Securitization and loan sales

Banks in our model hold mortgage loans on their books. How would securitization affect our results? The answer depends on how securitization impacts the aggregate risk exposure of banks to house prices. The shock to  $B_2$  is an *aggregate* shock impacting all consumers. It is therefore systemic and cannot be diversified away by securitization. So, even if banks securitize their loans, as long as mortgage-backed securities (MBS) stay within the banking system, our results will be unaffected. This seems to have been the case during the 2007–09 crisis, as banks held much of the outstanding volume of MBS (Acharya and Richardson, 2009). If, however, MBS are partly held by investors other than banks and the losses suffered by these investors do not affect banks' funding costs, then banks might lessen their exposure to house prices, weakening the correlated leverage result. But if non-bank investors holding MBS also provide funding to banks (say through repos involving MBS), the losses imposed by a negative housing market shock on those investors will weaken their ability to finance banks, thereby increasing banks' funding costs and amplifying the effect of the shock in the housing market. In this case, our results continue to hold.

#### 5.3.5. Multi-period loans

We assume that banks make single-period loans, so homeowners in each period sell their houses and repay their loans. Consider a multi-period setting in which homeowners repay loans partially over many periods before selling houses to repay the rest. A lower fraction of the loan repayment will depend on the selling prices of houses, but the dependence will be there nonetheless. Thus, a multi-period setting will not eliminate the impact of a fundamental housing market shock.

## 6. Extensions

We now examine various extensions. First, we introduce bank heterogeneity and examine the impact of too-big-to-fail (TBTF) protection for large banks. Second, we analyze how house prices affect the *depth* of the financial intermediation, and how this depth in turn affects house prices. Third, we analyze the feedback effects between housing market shocks and bank leverage.

### 6.1. Cross-sectional heterogeneity among banks and TBTF

Banks are *ex ante* identical in our base model. We now introduce heterogeneity: a fraction  $\alpha \in (0, 1)$  of banks are large and a fraction  $1 - \alpha$  are small. Regulators consider large banks TBTF, so these banks receive government assistance in the form of liquidity and/or equity infusions when there is a negative housing market shock, thereby obviating the need to raise additional funds to cover first-period deposit repayment shortfall at  $t = 2$ .<sup>28</sup> Small banks lack such “protection” and hence face a higher repayment-shortfall cost relative to large banks. We now have:

**Proposition 4.** *Large banks choose higher leverage in the first period than small banks. The volatility of the equilibrium second-period house price depends on the first-period leverage of small banks but not that of large banks, and the impact of the leverage choices of small banks on the second-period house price volatility is decreasing in the fraction of large banks,  $\alpha$ . The first-period leverage ratios of all banks increase as the fraction of large banks increases.*

This proposition can be understood as follows. TBTF protection has three consequences. First, by reducing the *ex post* cost of a deposit repayment shortfall for large banks, a TBTF policy encourages these banks to take on higher first-period leverage. Large banks' leverage choices thus do not affect the volatility of the equilibrium second-period house price. By contrast, small banks'

<sup>28</sup> In reality, large banks may only receive partial assistance to cover a fraction of the deposit repayment shortfall. Complete government assistance is assumed to simplify the mathematical analysis, and our results here are qualitatively the same under an alternative setup with partial assistance to large banks.

first-period leverage choice affects their deposit repayment shortfall when there is a negative shock to the housing market, which impacts their cost of lending and thus the second-period house price.

A second consequence is that large banks will extend more loans than small banks in the second period,<sup>29</sup> because they face a smaller marginal cost of lending than small banks after a first-period repayment shortfall. The lower lending by small banks partially arrests the second-period house price decline conditional on a negative shock, and thus reduces house price volatility. When the fraction of large banks ( $\alpha$ ) goes up, each small bank's second-period lending declines further, its marginal cost of lending falls, and house price volatility is lowered.

The third consequence is that the reduced house price volatility in the second period due to the increase in the fraction of large banks, in turn, causes small banks to increase their first-period leverage. This is because the adverse impact of first-period bank leverage on the second-period lending of small banks is diminished with an increase in the number of large banks, given that large banks cushion the effect of a negative housing-market shock on the second-period house price. Thus, TBTF protection not only causes large banks to choose higher leverage themselves, but by helping to increase the future house price, it also increases the small banks' leverage.<sup>30</sup>

## 6.2. Intermediation thinning

One traditional intermediation function of banks is to screen and discover the borrower's ability to generate income to repay the loan (e.g., Allen (1990), Coval and Thakor (2005), Millon and Thakor (1985), and Ramakrishnan and Thakor (1984)). In our previous analysis we sidestepped this issue by assuming that there is no uncertainty about the borrower's income, other than the proceeds from the house sale. We now extend the base model to study the bank's intermediation role.

### 6.2.1. House price and intermediation thinning

We add the following structure to examine how expectations of future house prices affect the bank's intermediation role. If a borrower does not fully repay her loan, the bank can claim a fraction  $\mu$  of her income, where  $\mu \in (0, 1)$  is a constant. However, each first-period consumer's income is her private information. The values of income for first-period consumers at  $t = 2$ ,  $X_1$ , are uniformly distributed on  $[0, \hat{X}]$ , where  $\hat{X}$  is a positive constant.<sup>31</sup> Since the measure of consumers,  $S_c$ , exceeds the supply of houses,  $S$ , there exists an income cutoff,  $X^* \equiv \hat{X}[S_c - S]/S_c$ , such that the measure of consumers with date-2 income  $X_1 \in [X^*, \hat{X}]$  is  $S$  (the high-income group), and the measure of consumers with date-2 income  $X_1 \in [0, X^*)$  is  $S_c - S$  (the low-income group). To simplify, we assume that, for each second-period consumer, the income at  $t = 3$ ,  $X_2 \geq 0$ , is non-stochastic.

Loan application is costless. If screening were precise, banks would only lend to the high-income group, since banks can claim more income from the high-income group in case of borrower default, which lowers the expected cost of bank deposits. However, screening is noisy, as explained below.

Banks specialize in pre-lending income screening at  $t = 1$ . A consumer with  $X_1 \in [X^*, \hat{X}]$  will be correctly identified to be in the high-income group and given credit. For a consumer with  $X_1 \in [0, X^*)$ , w.p.  $\xi \in [0, 1]$  a bank correctly identifies her as belonging to the low-income group and hence rejects her, and w.p.  $1 - \xi$  the bank mistakenly classifies the consumer as part of the high-income group and accepts her. For now, assume  $\xi$  is exogenously fixed.

<sup>29</sup> There is empirical evidence showing that banks that received rescue funds increased loan supply. For example, Li (2012) finds that banks receiving TARP (Troubled Asset Relief Program) funds during the recent crisis expanded their loan supply by 6.43% of total assets annually; TARP banks spent about one-third of their TARP capital to support new loans (and kept the rest to strengthen the balance sheet).

<sup>30</sup> In our model the ex ante leverage and risk choices of large banks do not worsen the crash in the bad state because banks correctly anticipate the extent of government assistance in the bad state and make their ex ante choices accordingly. Banks choose higher leverage ex ante precisely because they know that TBTF protection will dampen the drop in house price in the bad state. If the TBTF-protected banks irrationally overestimate their protection and take "excessive" risk in their lending, the house price crash in the bad state may worsen.

<sup>31</sup> Note the subscript in  $X_1$  represents the first-generation consumers but not the time of their income realization.

**Lemma 2.** Among those consumers whose loan applications are approved, a measure  $S$  of those with the lowest incomes will buy houses. The incomes of those consumers will be distributed on the support  $[0, X_{high}]$ , where

$$X_{high} = \widehat{X} \left[ 1 - \frac{[S_c - S][1 - \xi]}{S_c} \right], \tag{23}$$

which is increasing in the precision of bank screening,  $\xi$ . The bank’s posterior belief about the expected income of a homebuyer, denoted by  $X(\xi)$ , is:

$$X(\xi) = \frac{\widehat{X}}{2} \left[ \frac{S_c - S}{S_c} + \left[ 1 - \frac{[S_c - S][1 - \xi]}{S_c} \right] \left[ 1 - \frac{[S_c - S][1 - \xi]}{S} \right] \right], \tag{24}$$

which is also increasing in  $\xi$ .

To understand this lemma, note that consumers with lower incomes have less to lose when defaulting on their loans, and hence will behave more aggressively to buy houses (conditional on their loan applications being approved). When the precision of bank screening increases (higher  $\xi$ ), fewer consumers in the low-income group will be able to get loans. As a result, more consumers in the high-income group will be able to buy houses (larger  $X_{high}$ ) as they face less competition from low-income consumers, which, in turn, increases an average homebuyer’s income (larger  $X(\xi)$ ).

Suppose now that each bank can independently choose the precision  $\xi$  by investing  $c(\xi)$  in a screening technology. We assume  $c' > 0, c'' > 0$ , with the Inada conditions  $c'(0) = 0$  and  $c'(1) = \infty$ . The bank privately knows its precision,  $\xi$ , and the investment in the screening technology,  $c(\xi)$ .

When house prices are high, all the borrowers repay their loans in full. When house prices are low, the bank expects to get  $\mu X(\xi)$  from a borrower. Together with the proceeds from selling the foreclosed house, the bank receives  $P_{2l} + \mu X(\xi)$  per loan. As in the previous analysis, the payment received by the bank is assumed to be insufficient to repay depositors in full, i.e.,  $P_{2l} + \mu X(\xi) < D_1$ .<sup>32</sup>

We provide a comparative statics result pertaining to the equilibrium in this model.

**Proposition 5.** Compare two equilibria with different exogenous parameter values. The precision  $\xi$  with which banks screen is higher in the equilibrium in which the probability of a positive fundamental shock to the housing market,  $\theta$ , is lower. Moreover, bank and borrower leverage choices remain correlated in the presence of bank screening.

The intuition is that the bank’s credit risk depends on both the value of the house and the fraction  $\mu$  of the borrower’s income available for loan repayment. Thus, a higher expected future house price reduces the bank’s reliance on borrower income in collecting the loan repayment, which dilutes the bank’s pre-lending screening incentive. That is, intermediation “thins” as house prices rise. Moreover, the correlated leverage result is sustained in this extension.

6.2.2. Feedback from intermediation thinning to house price

We now show that intermediation thinning leads to even higher consumer leverage.

**Proposition 6.** The sensitivity of the equilibrium first-period house price to the probability of a positive fundamental housing-market shock,  $\partial P_1 / \partial \theta$ , is ceteris paribus higher when banks can endogenously choose the precision of screening ( $\xi$ ) than when this precision is exogenously fixed.

The intuition is as follows. We know from Proposition 5 that a higher expected future house price (higher  $\theta$ ) induces banks to lower their investment in screening (lower  $\xi$ ), leading to fewer rejections of low-income loan applications. Furthermore, low-income consumers behave more aggressively than high-income consumers in bidding for houses because they have less to lose in case of loan default (Lemma 2). Thus, the greater presence of low-income consumers in the housing market drives up

<sup>32</sup> Suppose counterfactually that this is not the case in equilibrium, so banks do not experience a deposit repayment shortfall. Then a marginal reduction in screening reduces screening costs, but has no effect on the expected cost of a deposit repayment shortfall. This means banks can increase profit by decreasing their investment in screening. However, this cannot be true in an equilibrium.

the first-period house price even further. An immediate consequence of this is that first-period leverage for consumers increases more in response to a positive fundamental shock in the housing market when banks endogenously choose screening precision than when they cannot.

Taken together, [Propositions 5 and 6](#) illustrate a channel through which a high expectation of future house price can induce banks to invest less in screening, and there is a feedback from this laxity in bank screening to housing demand. This feedback manifests itself in higher demand from low-income borrowers, resulting in more aggressive bidding for houses and thus a higher house price. This seems to be broadly consistent with what occurred during the 2007–09 crisis.

An interesting aspect of intermediation thinning is that it highlights the multiplicative ways in which risks are elevated when there is a high expectation of a positive housing market shock. With fixed screening precision, there is only one channel by which a high expectation of house price appreciation contributes to higher risk, and that is via an endogenous uptick in both bank and borrower leverage, and the resulting exacerbation of the downward pressure on house prices by the high leverage when there is a negative housing market shock. This aspect of the model is similar to the fire-sale models in [Shleifer and Vishny \(1992\)](#) and [Acharya and Viswanathan \(2011\)](#). However, when screening precision is chosen endogenously by banks, there is a second channel through which the housing market shock increases risk, and that is by causing banks to be less vigilant in screening borrowers.<sup>33</sup> Thus, when there is a high expectation of house price appreciation, intermediation thinning conspires with correlated leverage to cause lending standards to decline precisely when higher bank and borrower leverage have already increased system fragility.

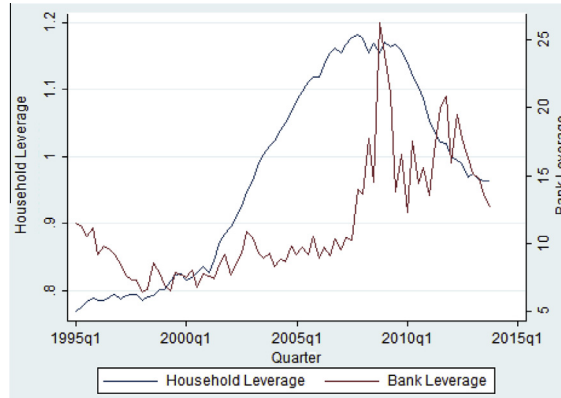
## 7. Empirical predictions, evidence, and policy implications

### 7.1. Predictions

First, the model's main prediction is that high house prices, high borrower leverage and high bank leverage occur together. The recent home mortgage crisis is an example of this. Second, there is a positive correlation between aggregate bank leverage in a given period and *subsequent* house price volatility. That is, house price volatility in a given period is decreasing in *appropriately-lagged* bank capital. In this section, we provide empirical support for these two predictions of the model.

Our model has two additional predictions whose tests are more challenging and conducting them would lead to a full-blown empirical paper, so we leave this for future research. Specifically, testing our result on bank interconnectedness (i.e., each bank's credit risk exposure is increasing in the past leverage choices of all other banks) must reckon with the fact that because bank leverage choices are endogenous (and in fact identical across banks in the model), a rigorous test will require instruments for leverage choices that are uncorrelated with credit risk exposure. Moreover, testing the model's result on intermediation thinning, wherein high house prices dilute bank screening, requires either a direct measure of bank screening, which is unobservable, or reliable indicators that can proxy for bank screening by measuring changes in variables that reflect changes in bank screening; see [Jayaraman and Thakor \(2014\)](#) for a recent example of such an indirect approach. Another hurdle is endogeneity – intermediation thinning also fuels higher house prices. However, there is some existing indirect evidence on intermediation thinning. The failure of the originate-to-distribute securitization model *only* during the recent subprime crisis (e.g., [Keys et al. \(2010\)](#) and [Purnanandam \(2011\)](#)), despite its long existence in the market, seems to suggest that, as our model predicts, it *might* be the high house prices during the recent crisis that caused banks to reduce their screening, thereby leading to the failure of the securitization model. On a related note, [Goetzmann et al. \(2012\)](#) find that, prior to the recent mortgage crisis, past home price appreciation increased the approval rate of subprime applications but did not affect the approval rate of prime applications, consistent with intermediation thinning and the consequent decline in the importance of borrower characteristics for the loan approval decision. Next, we provide evidence supporting the first two predictions.

<sup>33</sup> This aspect of the model has no relationship to the kinds of effects examined in the fire-sale literature.



**Fig. 3.** Household leverage and bank leverage (for commercial and investment banks) are plotted from 1995 to the first quarter of 2014. Household leverage is the ratio of the total liabilities of households and nonprofit organizations (available from Federal Flow of Funds data) to the annualized personal income available from the Bureau of Economic Analysis. The leverage of commercial and investment banks is calculated using data from Compustat and following [Hatzius \(2008, footnote 25\)](#).

## 7.2. Empirical evidence

Our first prediction already has existing empirical support. See [Fig. 2](#), which shows the behaviors of bank and consumer leverage and house prices prior to the subprime crisis. [Fig. 2\(a\)](#), adapted from [Hatzius \(2008\)](#), demonstrates the increase of aggregate leverage ratios of US banks (commercial and investment) from 2002 to 2008, where bank leverage is defined as total assets divided by bank equity capital. [Fig. 2\(b\)](#), adapted from [Mian and Sufi \(2011\)](#), shows that during the same time period household leverage, measured by the household debt-to-income ratio, also increased dramatically.<sup>34</sup> Finally, [Fig. 2\(c\)](#), also adapted from [Mian and Sufi \(2011\)](#), demonstrates strong house price appreciation for the large part of the 2002–08 time period.

We examine the robustness of this previously-documented relation between bank leverage and household leverage over a longer time period by calculating the leverage of commercial and investment banks using data from Compustat and following [Hatzius \(2008, footnote 25\)](#). We calculate household leverage as the ratio of the total liabilities of households and nonprofit organizations (available from Federal Flow of Funds data) to the annualized personal income available from the Bureau of Economic Analysis. We then estimate the correlation between bank leverage and household leverage during the 1995–2014 time period. [Fig. 3](#) shows that from 1995 to the first quarter of 2014, bank leverage and household leverage tended to move in the same direction. The coefficient of correlation between the two was 0.539.

Our second prediction – there is a positive correlation between aggregate bank leverage and subsequent house price volatility – does not have existing empirical support, so we test it.

**Data:** The data are drawn from several sources. Bank data (including bank leverage and size) come from the Call Report database from the Federal Financial Institutions Examination Council (FFIEC). All FDIC-insured financial institutions are required to file a consolidated report of condition and income (“call report”) to regulators on a quarterly basis, based on which we calculate bank leverage as  $\frac{\text{Total assets} - \text{Equity capital}}{\text{Total assets}}$ . For house prices, we use expanded-data home price indexes available from the Federal Housing Finance Agency (FHFA). We use seasonally adjusted quarterly indexes of house prices for all US states (including Washington, DC). Our analysis also uses data on state-level unemployment rates (from the Bureau of Labor Statistics), household income (from the US Census Bureau), and household debt (measured by mortgage debt balance per capita, extracted using a 5% random sample of the

<sup>34</sup> [Fig. 2\(b\)](#) also shows the behavior of corporate leverage during the same time period. As is evident, corporate leverage did not increase with bank leverage the way that consumer leverage did.

Consumer Credit Panel data from the Federal Reserve Bank of New York). Our sample period spans 2001–13.

**Empirical specifications and results:** Since house price volatility is not directly observable, we estimate it from house prices using two methods. In the first method (used in Models I–III), volatility is defined as the *absolute* value of the log-return of house prices over a one-quarter period in the future relative to the period in which bank capital is measured. In the second method (Model IV), we perform a maximum likelihood estimation using an ARCH model with a multiplicative heteroskedasticity specification. Taking quarter  $t$  as the measurement quarter for the independent variables, in Models I, II and III we measure volatility in quarter  $t$  for state  $i$  as the *absolute* value of the log-return of house prices in state  $i$  over a future one-quarter period,<sup>35</sup> ending in quarter  $t + 2$  (Model I),  $t + 3$  (Model II), or  $t + 4$  (Model III), and estimate the following OLS regression:

$$\text{House price absolute return}_{i,t+k-1 \text{ to } t+k} = \zeta_0 + \zeta_1 \times \text{Bank leverage}_{i,t} + \sum_j \zeta_j \times \text{Control } j + \epsilon_{i,t}, \quad (25)$$

where  $k \in \{2, 3, 4\}$ . The main independent variable is the leverage of banks headquartered in state  $i$ ,  $\text{Bank leverage}_{i,t}$ . We include several *controls*: (i) state unemployment rate, (ii) median household debt, (iii) median household annual income, and (iv) median size of banks headquartered in state  $i$ . We also control for the house price absolute return in the prior quarter and use quarter and state fixed effects.

The results are presented under Models I–III in Table 1. The coefficients on unemployment, household debt, bank leverage, and lagged house price absolute return are statistically significant with interpretable signs: absolute house price growth is lower when the unemployment rate is higher and households have more debt. Importantly, the coefficient on lagged bank leverage is positive and strongly statistically significant at the 1% level when the house price return is measured after three or four quarters, and significant at the 10% level when the house price return is measured after two quarters, consistent with the prediction of our model.<sup>36</sup>

The coefficients estimated in Models I–III capture the joint impact of the independent variables on house price growth and volatility. Our second empirical specification (Model IV), based on a maximum likelihood estimation, is a more sophisticated ARCH model in which house price growth and volatility are decoupled and *simultaneously* estimated using the following specification of multiplicative heteroskedasticity (with both quarter and state fixed effects):

$$\text{House price growth}_{i,t \text{ to } t+4} = \phi_0 + \sum_j \phi_j \times \text{Control } j + \epsilon_{i,t}, \quad (26)$$

where  $\text{House price growth}_{i,t \text{ to } t+4}$  is the percentage change in home price in state  $i$  from quarter  $t$  to quarter  $t + 4$ , and the controls included here are same as those in Eq. (25). The error terms  $\epsilon_{i,t}$  are independent and normally distributed with variance  $\sigma_{i,t}^2$  estimated by (with quarter fixed effects):

$$\text{Log}(\sigma_{i,t}^2) = \gamma_0 + \gamma_1 \times \text{Bank leverage}_{i,t} + \gamma_2 \times \text{Household debt}_{i,t}, \quad (27)$$

where  $\text{Household debt}_{i,t}$  is median household debt in state  $i$ . Our prediction states that the coefficient  $\gamma_1$  in (27) should be positive, i.e., future (four quarters ahead) house price volatility will be increasing (decreasing) in current bank leverage (bank capital ratio).

The results are presented under Model IV in Table 1. As is evident, the coefficient estimate on  $\text{Bank leverage}_{i,t}$  is positive and statistically significant at the 1% level. Taken together, the results of Models I through IV offer strong support for the second prediction of our model.

<sup>35</sup> Our results continue to hold when we use overlapping house price returns calculated over four-quarter periods.

<sup>36</sup> There may be other lags that reflect stronger or weaker relationships, but finding such lags would not alter the strength of the empirical support for our model, since our prediction is not about a specific lag structure but rather that there exists a lag for which bank capital and house price volatility are significantly related.

**Table 1**

Bank leverage and future house price volatility.

| Estimation procedure               | Model I                     | Model II              | Model III             | Model IV                                  |
|------------------------------------|-----------------------------|-----------------------|-----------------------|---|
|                                    | OLS                         | OLS                   | OLS                   | Maximum likelihood                        |
|                                    | House price absolute return |                       |                       | House price growth                        |
|                                    | $t + 1$ to $t + 2$          | $t + 2$ to $t + 3$    | $t + 3$ to $t + 4$    | $t$ to $t + 4$                            |
| Unemployment                       | −0.0017<br>(−4.64)***       | −0.0021<br>(−5.29)*** | −0.0022<br>(−5.31)*** | −0.0066<br>(−7.94)***                     |
| Household debt (\$ millions)       | −0.2165<br>(−2.25)**        | −0.3880<br>(−3.70)*** | −0.4959<br>(−4.57)*** | −9.4083<br>(−25.23)***                    |
| Household income (\$ millions)     | −0.1429<br>(−0.97)          | −0.0624<br>(−0.39)    | −0.1110<br>(−0.66)    | 0.7583<br>(1.72)*                         |
| Bank size (\$ billions)            | −0.0085<br>(−2.52)**        | −0.0087<br>(−2.37)**  | −0.0052<br>(−1.36)    | −0.0185<br>(−1.66)*                       |
| Bank leverage                      | 0.0243<br>(1.89)*           | 0.0368<br>(2.62)***   | 0.0405<br>(2.78)***   |   |
| Lagged house price absolute return | 0.5260<br>(22.20)***        | 0.3642<br>(14.09)***  | 0.2198<br>(8.20)***   |   |
| Constant                           | −0.0067<br>(−0.51)          | −0.0075<br>(−0.52)    | −0.0013<br>(−0.09)    | 0.2608<br>(9.84)***                       |
| Quarter fixed effects              | Yes                         | Yes                   | Yes                   | Yes                                       |
| State fixed effects                | Yes                         | Yes                   | Yes                   | Yes                                       |
|                                    |                             |                       |                       | <i>Log variance of house price growth</i> |
| Bank leverage                      |                             |                       |                       | 4.5862<br>(2.48)***                       |
| Household debt (\$ millions)       |                             |                       |                       | 85.4468<br>(15.24)***                     |
| Constant                           |                             |                       |                       | −13.3511<br>(−7.93)***                    |
| Quarter fixed effects              |                             |                       |                       | Yes                                       |
| Observations                       | 1407                        | 1407                  | 1407                  | 1407                                      |
| Log likelihood                     | 4585.75                     | 4463.27               | 4412.76               | 3000.98                                   |
| Adjusted $R^2$                     | .622                        | .5423                 | .491                  |   |
| Wald $\chi^2$                      |                             |                       |                       | 7582.76                                   |
| Degrees of freedom                 | 91                          | 91                    | 91                    | 89  |
| Probability                        |                             |                       |                       | 0.0000                                    |

Results of Models I, II, and III (Eq. (25)), and Model IV (Eqs. (26) and (27));  $t$  statistics are in parentheses.

\* Asterisks denote statistical significance at 10% levels.

\*\* Asterisks denote statistical significance at 5% levels.

\*\*\* Asterisks denote statistical significance at 1% levels.

### 7.3. Policy implications

The model highlights the role of banks in propagating and amplifying the effect of fundamental housing market shocks. While we assume no social cost of house price volatility, in practice regulators may perceive a cost – possibly because it increases consumption volatility for individuals – and may wish to reduce volatility. This can be done by diminishing the amplification channel by weakening the link between the financial conditions of banks and borrowers. Greater diversification of bank assets will reduce the sensitivity of house prices to localized fundamental shocks.

To see how this would work in the model, consider introducing a parameter,  $div$ , to our model, capturing the degree of diversification of banking across sectors and geographies. A larger  $div$  means that, conditional on a negative shock at  $t = 2$ , each bank's first-period deposit repayment shortfall is smaller, since diversification can partially offset its loss in the local housing market. Therefore, banks need to raise less funds in the second period. This lowers their marginal cost of lending, leading to a lower

loan rate. Consequently, the second-period consumers' housing demand increases, causing the house price  $P_{2l}$  at  $t = 2$  to decline less conditional on the negative shock.

A second policy implication is that house price volatility may be decreased by encouraging long-term home ownership. This can be done with mortgage-tax exemptions that increase with the home ownership duration. To see how this would work in the model, suppose the length of home ownership is  $T$  periods, so a fraction  $1/T$  of homes are traded in each period. Assume bank loans are repaid only when borrowers' homes are sold. The bank's profit in any period thus depends only on a fraction  $1/T$  of its loan portfolio. As  $T$  increases, a bank's deposit repayment shortfall and cost of new lending become less sensitive to housing market shocks in any period, thereby mitigating the amplifying effect of bank losses on house price volatility. Thus, long-term home ownership helps the bank to diversify across temporary housing market shocks. Moreover, the reduced housing supply in each period, due to a decrease in  $1/T$ , causes the equilibrium house price, conditional on a negative shock, to increase to clear the market, thereby reducing house price volatility.

A third policy implication is that time-varying, procyclical bank capital requirements can dampen house price volatility. Since banks increase leverage when borrowers are highly levered, regulators can link permissible borrower leverage to bank capital requirements against mortgages. That is, when bank capital requirements are higher, greater borrower leverage may be permitted in mortgages. This can be viewed both cross-sectionally – more highly-levered mortgages would require higher bank capital in Basel risk weights – or in an intertemporal sense that when capital requirements are ratcheted up, banks can extend mortgages to more highly-levered borrowers. Alternatively, holding fixed bank capital, imposing minimum-equity-input restrictions on home mortgages will have a similar effect. Recently, [Kumar and Skelton \(2013\)](#) have documented that requiring homeowners to provide sufficient equity has a significant impact on house price dynamics. They show that Texas, the only state with a regulation limiting mortgage debt to 80% of a home's market value, had a 1% decline in its house price index from 2007 to 2011, whereas this was a 20% decline nationally.

To illustrate the idea, consider the following numerical simulation, which shows that imposing capital adequacy rule on banks in our model reduces house price volatility and makes the banking system less fragile by lowering each bank's credit exposure.

**Data:** Suppose  $B_1 = 5.8, M_1 = 9, B_{2h} = 15, B_{2l} = 1, M_2 = 8, \beta P_{3h} = 48, \lambda = 0.82, \delta = 0.1, N = 53.3$ , and  $\theta = 0.85$ .

**Results:** Absent capital adequacy rule, wherein banks can freely choose their leverage ratios, a numerical analysis of our model yields  $P_1 = 21.47$  (first-period house price),  $P_{2h} = 31.29$  (second-period house price upon a positive shock), and  $P_{2l} = 8.21$  (second-period house price upon a negative shock). Each first-period homebuyer borrows  $L_1 = 12.47$  from a bank. To finance the loan, each bank raises equity capital  $E_1 = 1.40$  and deposits  $D_1 = 11.07$ . Therefore, the bank capital ratio is  $1.40/12.47 = 11\%$ . Each bank's credit exposure in the second period, conditional on a negative shock, is  $N(D_1 - P_{2l})^+ = 53.3 \times (11.07 - 8.21) = 152.44$ .

Now suppose a 13% capital requirement is imposed on all banks. Taking the loan size  $L_1 = 12.47$  as given, in the first period each bank will have to raise more equity  $E_1 = 1.62 > 1.40$ , and less deposits  $D_1 = 10.85 < 11.07$ . In this case, we can show that  $P_{2h}$  remains at 31.29, while  $P_{2l}$  rises to 10.43. That is, the second-period house price conditional on a negative shock is higher when banks are required to hold more capital in the first period. Consequently, house price volatility decreases, and each bank's credit exposure reduces to  $53.3 \times (10.85 - 10.43) = 22.39 < 152.44$ .

Finally, the intermediation thinning analysis suggests that bank screening quality declines during bull housing markets. If such deterioration leads to lower-quality bank asset portfolios that – perhaps due to interbank portfolio connectedness not modeled here – engender a higher probability of a financial crisis by ensnaring even those institutions not directly involved in real estate lending, then regulators would need to be especially vigilant during such times. Not only would bank capital requirements need to be higher when house prices are higher, but regulatory monitoring of forward-looking indicators of asset quality would also need to be stepped up.



**8. Conclusion**

We have developed a theoretical model explaining why bank leverage, consumer leverage and house prices tend to be positively correlated. The model produces numerous additional predictions, some of which we test and provide supporting empirical evidence for. It also generates several policy implications. These findings have implications for issues related to house price volatility and financial system fragility. Future research could extend the theoretical analysis to repeat homebuyers.

**Appendix**

**Explicit Expression Corresponding to Assumption 2:** The upper bound  $\widehat{B}$  is defined such that:

$$W_1 = Y + \frac{\delta Y + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} [Y - M_2]}{\lambda} + \frac{N}{\lambda} \left[ \delta Y + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} [Y - M_2] \right]^2 + \delta N Y^2, \tag{A1}$$

where  $W_1$  is defined in (A23), and

$$Y \equiv M_2 + \frac{\lambda + \delta}{2\lambda\delta N} \left[ \sqrt{1 + \frac{4\lambda\delta N}{\lambda + \delta} [\beta P_{3h} + \widehat{B} - M_2]} - 1 \right]. \tag{A2}$$

**Further discussions of Assumptions 1 and 2:** First, it can be shown the condition  $B_{2l} < \widehat{B}$  made in Assumption 2 ensures  $P_{2l} < D_{1j} \forall j$ , where  $P_{2l}$  is given by (A15) in the Proof of Lemma 1. Thus, we also have  $P_{2l} < R_1 L_1$ . Second, we know from the consumer’s indifference condition (7) that  $M_1 = B_1 + \mathbb{E}[(P_2 - R_1 L_1)^+] = B_1 + \theta(P_{2h} - R_1 L_1)^+ + [1 - \theta](P_{2l} - R_1 L_1)^+ = B_1 + \theta(P_{2h} - R_1 L_1)^+$ . Thus, the condition  $M_1 > B_1$  made in Assumption 1 ensures  $P_{2h} > R_1 L_1$ . Summarizing, Assumptions 1 and 2 ensure

$$P_{2l} < D_{1j} < R_1 L_1 < P_{2h} \forall j. \tag{A3}$$

**Proof of Lemma 1.** We solve for the second-period subgame equilibrium by taking the banks’ equilibrium first-period choices as given. In a symmetric equilibrium, each bank takes the same first-period capital structure and makes the same amount of loans (as will be shown explicitly in the proof of Proposition 3), so we denote  $D_{1j} = D_1$  and  $n_{1j} = n_1 \forall j$ . We first analyze the subgame in the second period when  $B_2 = B_{2h}$  is realized and hence  $n_{1j}(D_{1j} - P_{2h})^+ = 0 \forall j$  (see (A3)). In this case, the second-period symmetric subgame equilibrium is determined by the following equations (we drop the subscript  $j$  in equilibrium notations):

$$D_{2h} + E_{2h} = L_{2h}, \tag{A4}$$

$$\frac{D_{2h}}{E_{2h}} = \frac{\lambda}{\delta}, \tag{A5}$$

$$\beta R_{2h} L_{2h} - L_{2h} = \lambda n_{2h} E_{2h} L_{2h}, \tag{A6}$$

$$n_{2h} = N, \tag{A7}$$

$$M_2 = B_{2h} + \beta [P_{3h} - R_{2h} L_{2h}], \tag{A8}$$

where (A4)–(A8) come from substituting equilibrium outcomes in (8), (10), (12), (13) and (7), respectively. Solving this system of equations, we have the following unique solution:

$$P_{2h} = M_2 + \frac{\lambda + \delta}{2\lambda\delta N} \left[ -1 + \sqrt{1 + 4W_{2h} \frac{\lambda\delta N}{\lambda + \delta}} \right], \tag{A9}$$

where  $W_{2h} \equiv \beta P_{3h} + B_{2h} - M_2 = \beta R_{2h} L_{2h} > 0$  (from (A8)). It is clear that  $P_{2h}$  is increasing in  $B_{2h}$ .

Next, consider the case with  $B_2 = B_{2l}$  and hence  $n_{1j}(D_{1j} - P_{2l})^+ = n_{1j}[D_{1j} - P_{2l}] = n_1 [D_1 - P_{2l}] > 0 \forall j$  (see (A3)). The second-period symmetric subgame equilibrium is determined by the following equations:

$$D_{2l} + E_{2l} = L_{2l} + \frac{n_1}{n_{2l}} [D_1 - P_{2l}], \tag{A10}$$

$$\frac{D_{2l}}{E_{2l}} = \frac{\lambda}{\delta}, \tag{A11}$$

$$\beta R_{2l} L_{2l} - L_{2l} = \lambda n_{2l} E_{2l} \left[ L_{2l} + \frac{n_1}{n_{2l}} [D_1 - P_{2l}] \right], \tag{A12}$$

$$n_{2l} = N, \tag{A13}$$

$$M_2 = B_{2l} + \beta [P_{3h} - R_{2l} L_{2l}]. \tag{A14}$$

Together with  $n_1 = N$  and  $P_{2l} - L_{2l} = M_2$ , we solve the system of equations and get the unique solution:

$$P_{2l} = M_2 + W_{2l} - \frac{\lambda \delta N}{\lambda + \delta} [D_1 - M_2]^2, \tag{A15}$$

where  $W_{2l} \equiv \beta P_{3h} + B_{2l} - M_2 = \beta R_{2l} L_{2l} > 0$ . It is clear that  $P_{2l}$  is increasing in  $B_{2l}$ .

Since the overall (two-period) equilibrium must be subgame perfect, the unique second-period equilibrium (see (A9) and (A15) corresponding to  $B_{2h}$  and  $B_{2l}$ , respectively) must be part of the overall equilibrium. □

**Proof of Proposition 1.** When all banks make identical equilibrium choices in the first period, this proposition follows from the facts that  $P_{2h}$  in (A9) is not a function of  $D_1$ , while  $P_{2l}$  in (A15) is decreasing in  $D_1$  (note that  $D_1 > P_{2l} > M_2$ ). The proof can be easily extended to the more general case. □

**Proof of Proposition 2.** Let  $P_{2l}^I$  and  $L_{2l}^I$  be equilibrium outcomes in Equilibrium I for given first-period choices of all banks. Consider another equilibrium, II, in which each bank other than bank  $j$  uses greater first-period leverage than in Equilibrium I. We show the equilibrium price at  $t = 2$  when  $B_2 = B_{2l}$ ,  $P_{2l}^{II}$ , is less than  $P_{2l}^I$ . Suppose the contrary is true:  $P_{2l}^{II} \geq P_{2l}^I$ . Since all banks cannot extend less loans in Equilibrium II than in Equilibrium I, there is a bank, say bank  $j$ , making at least as many loans in Equilibrium II as in Equilibrium I. For this bank, the right-hand-side of (A10) is larger in Equilibrium II than in Equilibrium I, so  $D_{2l}^{II} + E_{2l}^{II} > D_{2l}^I + E_{2l}^I$ . Since the leverage ratio is fixed by (A11), it follows that  $D_{2l}^{II} > D_{2l}^I$  and  $E_{2l}^{II} > E_{2l}^I$ . Then, for bank  $j$  the right-hand-side of (A12) is greater in Equilibrium II than in Equilibrium I, while its left-hand-side is smaller in Equilibrium II than in Equilibrium I: a contradiction. □

**Proof of Proposition 3.** First, we derive the system of equations that determines the equilibrium in the first period. From (8)–(10), we can rewrite bank  $j$ 's second-period profit conditional on  $B_2 = B_{2l}$  as:

$$\pi_{2jl} = n_{2jl} [\beta R_{2l} L_{2l} - L_{2l}] - \frac{\delta \left\{ n_{2jl} \frac{\lambda}{\lambda + \delta} \left[ L_{2l} + \frac{n_{1j}}{n_{2jl}} [D_{1j} - P_{2l}] \right] \right\}^2}{2} - \frac{\lambda \left\{ n_{2jl} \frac{\delta}{\lambda + \delta} \left[ L_{2l} + \frac{n_{1j}}{n_{2jl}} [D_{1j} - P_{2l}] \right] \right\}^2}{2}. \tag{A16}$$

The bank's problem is to choose  $n_{2jl}$  to maximize  $\pi_{2jl}$  in (A16). Using the Envelope Theorem, we have:

$$\begin{aligned} \frac{\partial \pi_{2jl}}{\partial L_{1j}} &= -\delta n_{2jl} \frac{\lambda}{\lambda + \delta} \left[ L_{2l} + \frac{n_{1j}}{n_{2jl}} [D_{1j} - P_{2l}] \right] n_{1j} \frac{\lambda}{\lambda + \delta} - \lambda n_{2jl} \frac{\delta}{\lambda + \delta} \left[ L_{2l} + \frac{n_{1j}}{n_{2jl}} [D_{1j} - P_{2l}] \right] n_{1j} \frac{\delta}{\lambda + \delta} \\ &= -n_{1j} n_{2jl} \frac{\lambda \delta}{\lambda + \delta} \left[ L_{2l} + \frac{n_{1j}}{n_{2jl}} [D_{1j} - P_{2l}] \right], \end{aligned} \tag{A17}$$

and

$$\begin{aligned} \frac{\partial \pi_{2jl}}{\partial n_{1j}} &= -\delta n_{2jl} \frac{\lambda}{\lambda + \delta} \left[ L_{2l} + \frac{n_{1j}}{n_{2jl}} [D_{1j} - P_{2l}] \right] \frac{\lambda}{\lambda + \delta} [D_{1j} - P_{2l}] - \lambda n_{2jl} \frac{\delta}{\lambda + \delta} \left[ L_{2l} + \frac{n_{1j}}{n_{2jl}} [D_{1j} - P_{2l}] \right] \frac{\delta}{\lambda + \delta} [D_{1j} - P_{2l}] \\ &= -n_{2jl} \frac{\lambda \delta}{\lambda + \delta} \left[ L_{2l} + \frac{n_{1j}}{n_{2jl}} [D_{1j} - P_{2l}] \right] [D_{1j} - P_{2l}]. \end{aligned} \tag{A18}$$

Applying the equation  $P_{2l} - L_{2l} = M_2$  and the equilibrium loan market clearing conditions  $n_{1j} = n_{2jl} = N$  to (A17) and (A18) yields:

$$\frac{\partial \pi_{2jl}}{\partial D_{1j}} = -\frac{\lambda \delta N^2}{\lambda + \delta} [D_{1j} - M_2], \tag{A19}$$

$$\frac{\partial \pi_{2jl}}{\partial n_{1j}} = -\frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - M_2] [D_{1j} - P_{2l}]. \tag{A20}$$

Thus, the bank’s first-order condition for a first-period capital structure that maximizes the sum of its first-period and second-period expected profits,  $\pi_{1j} + \theta \pi_{2jh} + [1 - \theta] \pi_{2jl}$ , is  $\partial \pi_{1j} / \partial D_{1j} + [1 - \theta] \partial \pi_{2jl} / \partial D_{1j} = 0$  (note that  $\partial \pi_{2jh} / \partial D_{1j} = 0$  because the equilibrium conditions in (A4)–(A7) and (A8) are independent of  $D_{1j}$ ), which can be explicitly written as:

$$\lambda E_{1j} = \delta D_{1j} + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} [D_{1j} - M_2]. \tag{A21}$$

The first-order condition for the amount of first-period loans that maximizes  $\pi_{1j} + \theta \pi_{2jh} + [1 - \theta] \pi_{2jl}$  is  $\partial \pi_{1j} / \partial n_{1j} + [1 - \theta] \partial \pi_{2jl} / \partial n_{1j} = 0$  (note that  $\partial \pi_{2jh} / \partial n_{1j} = 0$  because the equilibrium conditions in (A4)–(A7) and (A8) are independent of  $n_{1j}$ ), which can be explicitly written as:

$$\theta R_1 L_1 - L_1 = \lambda N E_{1j}^2 + \delta N D_{1j}^2 + [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - M_2] [D_{1j} - P_{2l}]. \tag{A22}$$

The consumer’s indifference condition (7) can be explicitly written as:

$$\theta R_1 L_1 = \theta P_{2h} + B_1 - M_1 \equiv W_1 > 0, \tag{A23}$$

where  $P_{2h}$  is given by (A9). Thus, we can rewrite (A22) as:

$$W_1 - L_1 = \lambda N E_{1j}^2 + \delta N D_{1j}^2 + [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - M_2] [D_{1j} - P_{2l}]. \tag{A24}$$

Next, we prove the existence and uniqueness of the first-period equilibrium. From (A21), we have:

$$E_{1j} = \frac{\delta D_{1j} + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} [D_{1j} - M_2]}{\lambda}. \tag{A25}$$

Substituting this into (A24) yields:

$$\begin{aligned} \mathcal{Y}_D \equiv W_1 - D_{1j} - \frac{\delta D_{1j} + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} [D_{1j} - M_2]}{\lambda} - \frac{N}{\lambda} \left[ \delta D_{1j} + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} [D_{1j} - M_2] \right]^2 - \delta N D_{1j}^2 \\ - [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - M_2] [D_{1j} - P_{2l}] = 0, \end{aligned} \tag{A26}$$

where  $P_{2l}$  is given by (A15). Thus,  $D_{1j}$  exists and is given by the solution to the above equation. It is clear that:

$$\begin{aligned} \frac{\partial \mathcal{Y}_D}{\partial \theta} &= P_{2h} + \frac{\delta}{\lambda + \delta} [D_{1j} - M_2] + \left[ \delta D_{1j} + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} [D_{1j} - M_2] \right] \frac{2 \delta N}{\lambda + \delta} [D_{1j} - M_2] \\ &\quad + \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - M_2] [D_{1j} - P_{2l}] > 0, \end{aligned} \tag{A27}$$

and together with (A15) it can be shown that:

$$\frac{\partial \Upsilon_D}{\partial D_{1j}} = -1 - \frac{\delta + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta}}{\lambda} - \frac{2N}{\lambda} \left[ \delta D_{1j} + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} [D_{1j} - M_2] \right] \left[ \delta + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} \right] - 2\delta N D_{1j} - [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} \left[ [D_{1j} - P_{2j}] + [D_{1j} - M_2] \left[ 1 + \frac{2\lambda \delta N}{\lambda + \delta} [D_1 - M_2] \right] \right] < 0. \tag{A28}$$

Thus, using the Implicit Function Theorem we have:

$$\frac{\partial D_{1j}}{\partial \theta} = - \frac{\partial \Upsilon_D / \partial \theta}{\partial \Upsilon_D / \partial D_{1j}} > 0, \tag{A29}$$

and this monotonicity of  $D_{1j}$  with respect to  $\theta$  proves the uniqueness of  $D_{1j}$ . The existence and uniqueness of  $E_{1j}$  and  $L_1$  then result from (A21) and (A24). It is also clear from (A25) and (A26) that in a symmetric equilibrium, the ex ante identical banks choose the same first-period capital structure, which we denote as  $D_{1j} = D_1$  and  $E_{1j} = E_1 \forall j$ .

Finally, we prove the result on correlated leverage. Totally differentiating (A24) with respect to  $\theta$  yields:

$$P_{2h} - \frac{\partial L_1}{\partial \theta} = 2\lambda N E_{1j} \frac{\partial E_{1j}}{\partial \theta} + \left[ 2\delta N D_{1j} + 2[1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - M_2] \right] \frac{\partial D_{1j}}{\partial \theta} - \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - M_2] [D_{1j} - P_{2j}] + [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} \frac{\partial ([D_{1j} - M_1] [M_2 - P_{2j}])}{\partial D_{1j}} \frac{\partial D_{1j}}{\partial \theta}, \tag{A30}$$

which can be simplified as (using (A21)):

$$\frac{\partial L_1}{\partial \theta} = \frac{P_{2h} + \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - M_2] [D_{1j} - P_{2j}] - [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} \frac{\partial ([D_{1j} - M_1] [M_2 - P_{2j}])}{\partial D_{1j}} \frac{\partial D_{1j}}{\partial \theta}}{1 + 2\lambda N E_{1j}}. \tag{A31}$$

Note that totally differentiating (A26) with respect to  $\theta$ , we have:

$$P_{2h} > [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} \frac{\partial ([D_{1j} - M_2] [D_{1j} - P_{2j}])}{\partial D_{1j}} \frac{\partial D_{1j}}{\partial \theta} > [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} \frac{\partial ([D_{1j} - M_2] [M_2 - P_{2j}])}{\partial D_{1j}} \frac{\partial D_{1j}}{\partial \theta}. \tag{A32}$$

Thus, it is clear from (A31) that  $\partial L_1 / \partial \theta > 0$ , which is equivalent to showing that  $L_1 / M_1$  is increasing in  $\theta$ .

We now show that bank leverage  $D_{1j} / E_{1j}$  is also increasing in  $\theta$ . We can rewrite (A21) as:

$$\lambda = \left[ \delta + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} \right] \frac{D_{1j}}{E_{1j}} - [1 - \theta] \left[ \frac{\lambda \delta}{\lambda + \delta} \right] \frac{M_2}{E_{1j}}. \tag{A33}$$

Suppose when  $\theta$  increases,  $D_{1j} / E_{1j}$  decreases. In that case, the first term on the right-hand side (RHS) of (A33),  $\{\delta + [1 - \theta] \lambda \delta [\lambda + \delta]^{-1}\} [D_{1j} / E_{1j}]$ , decreases. So to keep the balance of the equation, the second term on the RHS,  $[1 - \theta] \lambda \delta [\lambda + \delta]^{-1} [M_2 / E_{1j}]$ , must decrease by the same amount. Thus, when the ratio  $M_2 / E_{1j}$  becomes larger, for the same degree of increase in  $\theta$ , the ratio  $D_{1j} / E_{1j}$  should decrease by a larger extent to ensure the RHS to be a constant. But we know that  $M_2 < D_{1j}$  (see (A3)), and when  $M_2$  approaches its upper limit  $D_{1j}$ ,  $D_{1j} / E_{1j} = \lambda / \delta$  becomes a constant that does not decrease with  $\theta$ . This shows that the ratio  $D_{1j} / E_{1j}$  cannot be decreasing in  $\theta$  and hence can only be increasing in  $\theta$ . Hence, what is left is to prove that  $M_2 / E_{1j}$  is increasing in  $M_2$  (so for  $\forall M_2 < D_{1j}$ , the ratio  $D_{1j} / E_{1j}$  can only be increasing in  $\theta$ ). To show this, we first rewrite (A21) as:

$$D_{1j} = \frac{\lambda E_{1j} + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} M_2}{\delta + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta}}. \tag{A34}$$

Substituting this into (A24), we have:

$$\begin{aligned} \mathcal{Y}_E \equiv & W_1 - \frac{\lambda E_{1j} + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} M_2}{\delta + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta}} - E_{1j} - \lambda N E_{1j}^2 - \delta N \left[ \frac{\lambda E_{1j} + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} M_2}{\delta + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta}} \right]^2 \\ & - [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} \left[ \frac{\lambda E_{1j} - \delta M_2}{\delta + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta}} \right] \left[ \left[ \frac{\lambda E_{1j} + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} M_2}{\delta + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta}} \right] + \frac{\lambda \delta N}{\lambda + \delta} \left[ \frac{\lambda E_{1j} - \delta M_2}{\delta + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta}} \right]^2 \right] \\ & + [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} [\beta P_{3h} + B_{2l}] = 0. \end{aligned} \tag{A35}$$

It is clear that  $\partial \mathcal{Y}_E / \partial E_{1j} < 0$ , and

$$\begin{aligned} \frac{\partial \mathcal{Y}_E}{\partial M_2} = & - \left[ 1 + 2\delta N D_{1j} + [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - P_{2l}] + [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - M_2] \right] \frac{[1 - \theta] \frac{\lambda \delta}{\lambda + \delta}}{\delta + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta}} \\ & + [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - P_{2l}] < - \frac{[1 - \theta] \frac{\lambda \delta}{\lambda + \delta}}{\delta + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta}} - \left[ 2\delta N + 2[1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} \right] \frac{[1 - \theta] \frac{\lambda \delta}{\lambda + \delta}}{\delta + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta}} \\ & - [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - P_{2l}] < 0. \end{aligned} \tag{A36}$$

We thus have  $\partial E_{1j} / \partial M_2 < 0$ , and hence it is clear that  $M_2 / E_{1j}$  is increasing in  $M_2$ . This completes the proof for the result that  $D_{1j} / E_{1j}$ , and hence  $D_1 / E_1$ , is increasing in  $\theta$ .  $\square$

**Proof of Proposition 4.** We focus on the analysis for the second-period equilibrium when  $B_2 = B_{2l}$ , using superscripts *LA* and *SM* to denote large and small banks, respectively. Following the analysis in the base model, we can derive the first-order conditions for a large bank’s profit-maximizing choices of second-period capital structure and loan amount:

$$\frac{D_{2jl}^{LA}}{E_{2jl}^{LA}} = \frac{\lambda}{\delta}, \tag{A37}$$

$$\beta R_{2l} - 1 = \lambda n_{2jl}^{LA} \frac{\delta}{\lambda + \delta} L_{2l}, \tag{A38}$$

and the corresponding first-order conditions for a small bank:

$$\frac{D_{2jl}^{SM}}{E_{2jl}^{SM}} = \frac{\lambda}{\delta}, \tag{A39}$$

$$\beta R_{2l} - 1 = \lambda n_{2jl}^{SM} \frac{\delta}{\lambda + \delta} \left[ L_{2l} + \frac{n_{1j}^{SM}}{n_{2jl}^{SM}} [D_{1j}^{SM} - P_{2l}] \right]. \tag{A40}$$

To clear the second-period loan market, we must have:

$$\alpha n_{2jl}^{LA} + [1 - \alpha] n_{2jl}^{SM} = N, \tag{A41}$$

$$n_{2jl}^{SM} L_{2l} + n_{1j}^{SM} [D_{1j}^{SM} - L_{2l} - M_2] = n_{2jl}^{LA} L_{2l}, \tag{A42}$$

where (A42) comes from (A38) and (A40). Thus,

$$n_{2jl}^{SM} = N - \alpha \frac{n_{1j}^{SM} [D_{1j}^{SM} - L_{2l} - M_2]}{L_{2l}} < N, \tag{A43}$$

$$n_{2jl}^{LA} = N + [1 - \alpha] \frac{n_{1j}^{SM} [D_{1j}^{SM} - L_{2l} - M_2]}{L_{2l}} > N. \tag{A44}$$

From the consumer’s indifference function, we have  $\beta R_{2l} L_{2l} = W_{2l} \equiv \beta P_{3h} + B_{2l} - M_2$ . Substituting this and (A44) into (A38), we have:

$$r_{TB} \equiv W_{2l} - L_{2l} - \frac{\lambda\delta}{\lambda + \delta} \left\{ NL_{2l}^2 + [1 - \alpha]L_{2l}n_{1j}^{SM} [D_{1j}^{SM} - L_{2l} - M_2] \right\} = 0. \tag{A45}$$

It is clear that:

$$\frac{\partial L_{2l}}{\partial D_{1j}^{SM}} = - \frac{\frac{\lambda\delta}{\lambda + \delta} [1 - \alpha]L_{2l}n_{1j}^{SM}}{1 + \frac{\lambda\delta}{\lambda + \delta} \left\{ 2L_{2l} [N - [1 - \alpha]n_{1j}^{SM}] + [1 - \alpha]n_{1j}^{SM} [D_{1j}^{SM} - M_2] \right\}} < 0, \tag{A46}$$

and

$$\frac{1}{\left| \partial L_{2l} / \partial D_{1j}^{SM} \right|} \propto \frac{1 + \frac{\lambda\delta}{\lambda + \delta} 2NL_{2l}}{1 - \alpha} + \frac{\lambda\delta}{\lambda + \delta} n_{1j}^{SM} [D_{1j}^{SM} - 2L_{2l} - M_2], \tag{A47}$$

and hence  $\left| \partial L_{2l} / \partial D_{1j}^{SM} \right|$  is decreasing in  $\alpha$ .

Next, we show that large banks choose higher leverage in the first period. Following the analysis in the base model, we can derive the first-order conditions for profit-maximizing first-period capital structures for large and small banks:

$$\lambda E_{1j}^{LA} = \delta D_{1j}^{LA}, \tag{A48}$$

$$\lambda E_{1j}^{SM} = \delta D_{1j}^{SM} + [1 - \theta] \frac{\lambda\delta}{\lambda + \delta} \left[ \frac{n_{2jl}^{SM}}{n_{1j}^{SM}} L_{2l} + [D_{1j}^{SM} - P_{2l}] \right]. \tag{A49}$$

It is thus clear that:

$$\frac{D_{1j}^{LA}}{E_{1j}^{LA}} = \frac{\lambda}{\delta} > \frac{D_{1j}^{SM}}{E_{1j}^{SM}}. \tag{A50}$$

Finally, we show the effect of the presence of large banks on the small banks' first-period leverage ratios. From the Proof of Proposition 3, we know that:

$$\begin{aligned} \frac{\partial \pi_{2jl}^{SM}}{\partial D_{1j}^{SM}} &= -n_{1j}^{SM} n_{2jl}^{SM} \frac{\lambda\delta}{\lambda + \delta} \left[ L_{2l} + \frac{n_{1j}^{SM}}{n_{2jl}^{SM}} [D_{1j}^{SM} - P_{2l}] \right], \\ &= -n_{1j}^{SM} \frac{\lambda\delta}{\lambda + \delta} [NL_{2l} + [1 - \alpha]n_{1j}^{SM} [D_{1j}^{SM} - P_{2l}]], \end{aligned} \tag{A51}$$

which is increasing in  $\alpha$ . It follows that  $\partial \pi_{1j}^{SM} / \partial D_{1j}^{SM} + [1 - \theta] \partial \pi_{2jl}^{SM} / \partial D_{1j}^{SM}$  is increasing in  $\alpha$  for a small bank's optimal first-period capital structure choice. Suppose  $D_{1j}^{SM}$  is the solution for the first-order condition for an optimal first-period capital structure, such that  $\partial \pi_{1j}^{SM} / \partial D_{1j}^{SM} + [1 - \theta] \partial \pi_{2jl}^{SM} / \partial D_{1j}^{SM} = 0$ , for some  $\alpha$ . Consider some  $\alpha' > \alpha$ , in which case  $\partial \pi_{1j}^{SM} / \partial D_{1j}^{SM} + [1 - \theta] \partial \pi_{2jl}^{SM} / \partial D_{1j}^{SM} > 0$  (since the left-hand side is increasing in  $\alpha$ ). Thus, the optimal solution in the case with  $\alpha'$ ,  $D_{1j}^{SM'}$ , must be greater than  $D_{1j}^{SM}$ .  $\square$

**Proof of Lemma 2.** We first show low-income consumers behave more aggressively to buy houses. Consider a consumer whose loan application is approved. If she does not buy a house, her expected utility is:

$$U_1^{nb} = M_1 + X_1, \tag{A52}$$

and her expected utility from buying a house is:

$$U_1^b = B_1 + \theta [P_{2h} - R_1 L_1 + X_1] + [1 - \theta] [1 - \mu] X_1. \tag{A53}$$

Note that:

$$U_1^b - U_1^{nb} = B_1 + \theta [P_{2h} - R_1 L_1] - M_1 - [1 - \theta] [\mu X_1], \tag{A54}$$

which is decreasing in  $X_1$ . This proves the first part of the lemma.

Next, we derive  $X_{high}$ . Note that a loan application by any consumer in the low-income group will be approved w.p.  $1 - \xi$ . Thus, by law of large numbers, exactly  $1 - \xi$  fraction of consumers in the low-income group will be able to get a loan. To clear the housing market, we have:

$$[S_c - S][1 - \xi] + S[X_{high} - X^*][\widehat{X} - X^*]^{-1} = S, \tag{A55}$$

which yields (23).

Finally, we derive  $X(\xi)$ . Note that:

$$\frac{\Pr(X_1 \in [0, X^*])}{\Pr(X_1 \in [X^*, X_{high}])} = \frac{X^*[1 - \xi]}{X_{high} - X^*} = \frac{[S_c - S][1 - \xi]}{S - [S_c - S][1 - \xi]}, \tag{A56}$$

which yields:

$$\Pr(X_1 \in [X^*, X_{high}]) = \frac{S - [S_c - S][1 - \xi]}{S}, \tag{A57}$$

$$\Pr(X_1 \in [0, X^*]) = \frac{[S_c - S][1 - \xi]}{S}. \tag{A58}$$

Thus, we have:

$$\begin{aligned} X(\xi) &= \Pr(X_1 \in [0, X^*]) \frac{X^*}{2} + \Pr(X_1 \in [X^*, X_{high}]) \frac{X^* + X_{high}}{2} \\ &= \frac{\widehat{X}}{2} \left[ \frac{S_c - S}{S_c} + \left[ 1 - \frac{[S_c - S][1 - \xi]}{S_c} \right] \left[ 1 - \frac{[S_c - S][1 - \xi]}{S} \right] \right], \end{aligned} \tag{A59}$$

which is clearly increasing in  $\xi$ . □

**Proof of Proposition 5.** The bank's optimization problem in the first period in this case with screening is to choose its first-period capital structure ( $D_{1j}$  and  $E_{1j}$ ), the number of loans to make ( $n_{1j}$ ) and screening precision ( $\xi$ ) to maximize the following objective function:

$$\pi_{1j} + \theta\pi_{2hj} + [1 - \theta]\pi_{2jl} - c(\xi), \tag{A60}$$

where  $\pi_{1j}$  is given by (14) and  $\pi_{2jh}$  and  $\pi_{2jl}$  are given by (9) with:

$$D_{2jh} + E_{2jh} = L_{2h}, \tag{A61}$$

$$D_{2jl} + E_{2jl} = L_{2l} + \frac{n_{1j}}{n_{2jl}} [D_{1j} - \mu X(\xi) - P_{2l}]. \tag{A62}$$

Following the Proof of Proposition 3, it can be shown that

$$\frac{\partial \pi_{2jl}}{\partial \xi} = \frac{\lambda \delta N^2}{\lambda + \delta} [D_{1j} - \mu X(\xi) - M_2] \mu X'(\xi) > 0. \tag{A63}$$

The first-order condition for a profit-maximizing choice of bank screening precision is:

$$\mathcal{Y}_\xi \equiv [1 - \theta] \frac{\lambda \delta N^2}{\lambda + \delta} [D_{1j} - \mu X(\xi) - M_2] \mu X'(\xi) - c'(\xi) = 0. \tag{A64}$$

Thus,

$$\frac{\partial \xi}{\partial \theta} = - \frac{\partial \mathcal{Y}_\xi / \partial \theta}{\partial \mathcal{Y}_\xi / \partial \xi} = - \frac{- \frac{\lambda \delta N^2}{\lambda + \delta} [D_{1j} - \mu X(\xi) - M_2] \mu X'(\xi)}{\partial \mathcal{Y}_\xi / \partial \xi} < 0, \tag{A65}$$

where the last inequality comes from the fact that  $\partial \mathcal{Y}_\xi / \partial \xi < 0$ , which is the second-order condition and is negative as ensured by the Inada condition satisfied by  $c(\xi)$ .

Next, we show the correlated leverage result. Following the Proof of Proposition 3, it can be shown that:

$$\frac{\partial \pi_{1j}}{\partial D_{1j}} = -\frac{\lambda \delta N^2}{\lambda + \delta} [D_{1j} - \mu X(\xi) - M_2], \tag{A66}$$

$$\frac{\partial \pi_{2j}}{\partial n_{1j}} = -\frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - \mu X(\xi) - M_2][D_{1j} - \mu X(\xi) - P_{2j}], \tag{A67}$$

and the bank’s first-order condition for  $D_{1j}$  is:

$$\lambda E_{1j} = \delta D_{1j} + [1 - \theta] \frac{\lambda \delta}{\lambda + \delta} [D_{1j} - \mu X(\xi) - M_2], \tag{A68}$$

and the bank’s first-order condition for  $n_{1j}$  is:

$$\theta R_1 L_1 - L_1 = \lambda N E_{1j}^2 + \delta N D_{1j}^2 + [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - \mu X(\xi) - M_2][D_{1j} - \mu X(\xi) - P_{2j}]. \tag{A69}$$

The result on correlated leverage can be shown by replacing  $M_2$  and  $P_{2j}$  in the Proof of Proposition 3 with  $\mu X(\xi) + M_2$  and  $\mu X(\xi) + P_{2j}$ , respectively. □

**Proof of Proposition 6.** We know from the Proof of Lemma 2 that  $U_1^b - U_1^{nb} = 0$  for consumers with  $X_1 = X_{high}$ , where  $X_{high}$  is given by (23). That is,

$$\theta R_1 L_1 = W_1 - [1 - \theta][\mu X_{high}]. \tag{A70}$$

Recall  $W_1 \equiv \theta P_{2h} + B_1 - M_1$ . Substituting the above equation into (A69) yields:

$$W_1 - [1 - \theta]\mu X_{high} - L_1 = \lambda N E_{1j}^2 + \delta N D_{1j}^2 + [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - \mu X(\xi) - M_2][D_{1j} - \mu X(\xi) - P_{2j}]. \tag{A71}$$

Consider two cases: (i) the precision of bank screening,  $\xi$ , is exogenously fixed, and (ii) banks can choose  $\xi$ . We analyze the two cases, starting with the same set of parameter values, so  $\xi$  and  $X_{high}$  are the same across the two cases. First, consider case (i), where  $\xi$  is fixed, so  $X_{high}$  is also fixed. Following the Proof of Proposition 3, we can show that:

$$\frac{\partial L_1}{\partial \theta} = \frac{P_{2h} + \mu X_{high} + \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - M_2][D_{1j} - P_{2j}] - [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} \frac{\partial (D_{1j} - M_1)[M_2 - P_{2j}]}{\partial D_{1j}} \frac{\partial D_{1j}}{\partial \theta}}{1 + 2\lambda N E_{1j}}. \tag{A72}$$

In case (ii), we have:

$$\frac{\partial L_1}{\partial \theta} = \frac{P_{2h} + \mu X_{high} + \frac{\lambda \delta N}{\lambda + \delta} [D_{1j} - M_2][D_{1j} - P_{2j}] - [1 - \theta] \frac{\lambda \delta N}{\lambda + \delta} \frac{\partial (D_{1j} - M_1)[M_2 - P_{2j}]}{\partial D_{1j}} \frac{\partial D_{1j}}{\partial \theta}}{1 + 2\lambda N E_{1j}} - [1 - \theta] \mu \frac{\partial X_{high}}{\partial \theta}. \tag{A73}$$

Note that  $X_{high}$  is increasing in  $\xi$  (Lemma 2), but  $\xi$  is decreasing in  $\theta$  (see Proposition 5). Thus,  $\partial X_{high} / \partial \theta < 0$ , and hence  $\partial L_1 / \partial \theta$  is larger in case (ii) than in case (i) for the same set of exogenous parameter values. □

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