IRRATIONALITY, ASSET PRICING, AND FINANCIAL INTERMEDIARIES

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ABSTRACT

This paper shows that when irrational investors can impact prices to cause predictability in returns, and rational investors are wealth constrained in their ability to arbitrage mispricing, a financial intermediary may arise as an attempt by rational investors to relax wealth constraints. The financial intermediary actively trades on behalf of investors in exchange for a fee. Surprisingly, financial intermediation is facilitated by the learning ability of irrational investors. Irrational investors learn about the superior trading profits of rational investors but confuse their rationality with superior information. That is, they delegate the intermediary to invest on their behalf because they think it has better information, even though the source of its trading profits is its rationality. This helps the intermediary mitigate the predictability in prices the irrational investors create in the first place, despite the fact that it possesses no informational advantage over anybody. The intermediary does not eliminate equilibrium mispricing because it strategically limits the size of its funds to maximize its profit. A large financial intermediary is shown to be more profitable than multiple small intermediaries. Empirical implications are drawn about the profitability of intermediaries, predictability in prices, and wealth transfers between rational and irrational investors.

Irrationality, Asset Pricing, and Financial Intermediaries

1. INTRODUCTION

The mounting evidence on mispricing and 'anomalies' has motivated financial economists to relax the assumption that all investors are rational and recognize that investor psychology may play an important role in asset pricing. For example, some of the most bothersome anomalies that seem difficult to explain with "rational" asset pricing models are the predictability of market returns based on price to fundamental ratios even after controlling for risk, closed-end fund discount, short-term momentum and long-term reversal in market returns, equity premium puzzle, and risk-free rate puzzle.[1] By contrast, models in which such irrational behavior survives in the aggregate – often called "behavioral" asset pricing models – produce predictions that seem more consistent with documented patterns in financial data.[2]

Since it is unlikely that *all* agents would be irrational, many behavioral asset pricing models allow rational agents to co-exist with irrational agents. This is important to address the suspicion that rational agents *could* negate the price impact of their irrational counterparts. The main result in such a setting is that the ability of rational investors to exploit profitable opportunities created by the trading of irrational investors is limited, either because of the relative size of the rational agents (for example, Barberis et al. (1998) and Daniel et al. (1998)) or their risk aversion (for example, Daniel et al. (2001), DeLong et al. (1990), Hirshleifer and Luo (2001), and Hong and Stein (1999)).

Thus, rational investors are viewed as facing constraints on their arbitrage ability. However, since these models confine the interaction between rational and irrational investors to *market-based trading*, it is *not* clear how robust their results are to mechanisms that may allow these investors to interact and relax idiosyncratic constraints on their arbitrage ability. The particular mechanism I have in mind is the ability of rational and irrational investors to enter into contracts. Contracts with payoffs contingent on prices and trading outcome may affect the trading strategies of investors and thereby

[1]Hawawini and Keim (1995) and Hirshleifer (2001) survey some of this evidence.

[2]See Hirshleifer (2001) for an excellent review. Shleifer (2000) explains some of these theories.

also the inefficiency in prices. Similar contracts are commonly observed between financial intermediaries and their investors. Thus, one can visualize financial intermediaries assisting rational investors in profiting from the price impact of the irrational investors. This suggests that there is a significant gap in the existing literature in that rational intermediaries are not included along with irrational agents to examine equilibrium price patterns when intermediaries may possess the ability to arbitrage away the price impact of the irrational agents.

The purpose of this research is to attempt to fill this gap in literature. Specifically, the questions I address are:

(1) In a market in which there are both rational and irrational agents, but the aggregate trading wealth of the irrational agents is large enough to ensure that these agents have price impact, what are the incentives for the *endogenous* emergence of a financial intermediary to arbitrage away this price impact?
(2) Will the presence of an endogenously-arising financial intermediary eliminate the price impact of irrational investors?

To answer these questions, I develop a model in which irrational investors introduce predictability in returns, and wealth constraints limit the ability of a rational investor to profit from the predictability. The irrational investors overestimate the precision of the information contained in a *public signal* about the final payoff of a risky security. That is, they are overconfident. Overconfidence has been widely documented as a robust behavioral irrationality (De Bondt and Thaler (1995), Odean (1998)) and has been studied extensively in behavioral finance (for example, Daniel et al. (1998, 2001), Kyle and Wang (1997)).[3]

All investors have limited wealth but the number of irrational investors is large so their aggregate wealth is large. There is one *rational investor* who correctly interprets the public signal. In addition, there are liquidity investors who trade for exogenous reasons. The rational investor combines his order with those of liquidity investors so that the price-setting market maker (a coalition of irrational investors) observes only the

[3]While I focus on overconfidence, my model is general enough to accommodate other forms of irrationality.

aggregate order flow. Since irrational investors do not acknowledge their irrationality, they learn nothing from the rational investor's trade and do not update their beliefs conditional on the order flow. Being overconfident, they overestimate the information in the public signal and set an equilibrium price that overreacts to the public signal. The rational investor exploits this mispricing, but his trading profit is constrained by his personal wealth.

I then show that in a static model, the rational investor is unable to increase his trading profit by raising external finance from irrational investors. The reason is that the terms at which the irrational investors provide funds (as debt or equity) are unfavorable to the rational investor because the irrational investors do not expect him to trade more profitably than they do. This also rules out intermediation in a static model.

But suppose now that the irrational investors believe that some investor may receive a *private signal* about the payoff of the risky security in addition to the public signal, while continuing to assign zero probability to the event that there is somebody more rational than they are. They, however, use Bayes rule to learn about the existence or the quality (information content) of the potential private signal.

In this setting, I can address the question about the endogenous emergence of an intermediary. The rational investor creates a financial intermediary and offers to trade profitably on behalf of irrational investors. This intermediary borrows funds from investors, trades using these funds, returns the proceeds, and charges a fee for this service. The irrational investors invest with the intermediary if its fee does not exceed the profit that they expect it to make. Thus, the intermediary's perceived trading advantage determines the size of its fee.

With repeated trading, irrational investors update their beliefs by observing the trading profit of the intermediary. The intermediary earns greater expected profit on average than they do. Since irrational investors do not think of themselves as irrational, they assume that the source of intermediary's consistently-superior trading performance is the superior information of its private signal, and they revise upwards their estimate of the quality of this signal. As this estimate increases, the intermediary can charge a fee large enough to exceed the profit the rational investor could have made trading on his account with limited personal wealth.

In answer to the second question, I find that the intermediary does not eliminate equilibrium mispricing. The reason is that it endogenously imposes an "overfishing" constraint on itself, resulting in a self-imposed limit on the size of the funds it raises from investors. This allows it to limit the aggressiveness of its trading and the consequent price revelation, so as to maximize its trading profit.

Irrational investors invest in the intermediary and also trade on their own account. Their investment choices are rational given their beliefs. Bayesian learning by irrational investors causes their estimate of the quality of the intermediary's information to converge to a limiting value determined by the degree of their irrationality. If investors are more irrational, the rational intermediary appears to them to have a greater advantage over them. The average mispricing decreases as the size of the intermediary increases.

In an extension of the model to *multiple* rational investors, I find that the rational investors can maximize their expected profits by colluding to form a large financial intermediary rather than multiple competing intermediaries. When financial intermediaries formed by rational investors compete, the aggregate profit of the intermediaries and the mispricing decrease in the number of competing financial intermediaries.

I draw several empirical predictions. First, irrational investors lose money to rational investors if there is mispricing. I show that in illiquid markets, the transfer of wealth from irrational to rational investors is small even with significant mispricing. Thus, anomalous (predictable) price behavior is more likely to persist in such markets. Second, the strength of financial anomalies will decrease as the financial intermediation sector grows in an economy. Third, if investors are overconfident, the intermediaries are more likely to be contrarian investors. Fourth, financial intermediation is more profitable when (anomalous) predictability in prices is high.

To assess the marginal contribution of my work, it is useful to divide the existing literature into three categories. The first category explains price anomalies using models of irrational individuals. Daniel, Hirshleifer, and Subrahmanyam (2001) show that price impact of overconfident investors can cause expected return to depend on risk as well as on measures of mispricing such as book to market ratio. Barberis, Shleifer, and Vishny

(1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999) explain overreaction and underreaction by modeling imperfectly rational investors.[4] My model is consistent with this literature in that overconfident investors induce overreaction, and my purpose is not to provide an alternative theory of price patterns.

A natural question is what happens if there are rational investors. Will they not drive out the irrational investors? De Long et al. (1990, 1991) and Hirshleifer and Luo (2001) confront this issue to show that irrational investors may earn higher expected return (but not utility) than rational investors by trading aggressively and taking greater risk. Thus, risk aversion constrains the arbitrage ability of rational investors in these models.[5] By contrast, I model risk-neutral investors and, in the static model, the rational investor has *no* price impact at all.

All of these papers leave an important issue unattended. The classical argument in favor of market efficiency does not require that all investors be rational; efficiency can result even with some irrational agents as long as they do not have aggregate price effects in equilibrium (Fama (1965)). Alchian (1950) and Friedman (1953) assert that those who behave irrationally will be driven out of the market by those who behave rationally. The fact that individual investors are constrained in such arbitrage raises the question of whether financial intermediaries are the effective arbitrageurs. If intermediaries can arbitrage and eliminate mispricing, then behavioral theories of anomalies are on shaky ground. Shleifer and Vishny (1997) address this issue with exogenous intermediaries who engage in professional arbitrage using borrowed capital. They show that arbitrage opportunities may not be completely eliminated in this setting because investors who

[4]In Daniel et al. (1998), overconfident investors overreact to information causing price changes that reverse subsequently. If they update overconfidence over time, overconfidence can aggravate temporarily causing short-lag positive autocorrelation in returns. In Barberis et al. (1998), investors with incorrect beliefs about the model of earnings underreact to a short spell of good earnings and overreact to a long spell of good earnings. Hong and Stein (1999) show that 'newswatchers' who do not learn from price cause underreaction and momentum traders' imperfect arbitrage leads to overreaction.

[5]Hong and Stein (1999) and Daniel et al. (2001) also show that risk averse rational investors can attenuate but not eliminate mispricing. Kyle and Wang (1997) and Benos (1998) provide a different perspective. In models with rational pricing, they show that overconfidence acts as a commitment device and yields overconfident investors greater profit than rational investors.

finance arbitrageurs may withdraw their funds when arbitrage opportunities are most valuable. However, the arbitrageur in this model as well as the withdrawal behavior of investors are exogenously imposed as mechanisms to illustrate how attempts to eliminate arbitrage may be limited in effectiveness. Thus, investors don't learn and one cannot introduce contracts that could potentially overcome the arbitrageur's constraints. By contrast, my main contribution is to specify the beliefs of different agents, allow learning, and *endogenously* derive the contracts that permit intermediaries to be profitable. I show that intermediation leads to a wealth transfer from irrational to rational investors. Yet, mispricing is not entirely eliminated.[6] I also derive empirical implications about the profitability of intermediaries and the degree of mispricing.[7]

The rest of this chapter is organized as follows. Section 2 sets up the basic model and trading mechanism when all investors are irrational and characterizes mispricing. Section 3 shows how a rational investor trading on its own account attempts to exploit mispricing. Section 4 shows how a rational investor can leverage its advantage by forming financial intermediary when irrational investors learn in a confused way. Section 5

[6]There are two reasons. The first is similar to that in Shleifer and Vishny, namely that irrational investors misestimate the profitability of arbitrage and they sometimes provide more and sometimes less funds than are needed to eliminate arbitrage. The second reason is that in my model maximizing arbitrage profit is not synonymous with eliminating mispricing. Even when it is not rationed, the intermediary imposes a limit on aggressiveness of its arbitrage strategy to maximize its trading profit.

[7]Theories of financial intermediation include Diamond (1984), who provides a theory of banks based on minimizing the costs of delegated monitoring of borrowers, and Ramakrishnan and Thakor (1984), who show that information brokers can improve welfare by minimizing the costs of information production and moral hazard. Both papers show that diversification makes it optimal to have an infinitely large financial intermediary. By contrast, in my analysis, there is no difference in social welfare between having one large intermediary and having multiple small intermediaries. However, a large financial intermediary can earn greater income than the combined income of many small financial intermediaries, so there is an income-motive to being large. The financial intermediaries most similar to what I model are mutual funds or hedge funds. My formulation captures the popular notion that hedge funds help their investors exploit mispricing that they themselves cannot exploit. The size of the intermediary's trading position in my analysis is determined by a tradeoff between the greater exploitation of mispricing and the revelation of information through aggressive trading. This appears to be an important feature of real life hedge funds; see Brown, Goetzmann, and Ibbotson (1999).

discusses extensions and empirical implications. Section **??** concludes. All proofs are in the Appendix.

2. BASIC MODEL

I first consider a standard model of informed trading with the variation that all investors are irrational in an identical way. The purpose of this section is to lay out the basic framework and also derive some microstructure results to be used in subsequent sections. There are 5 dates in the model: 0, 1, 2, 3, and 4. The following subsections explain the model in detail.

A. **Securities.** There is a riskless security and a risky security. The riskless security, measured in dollars, is the numeraire and the consumption good. The risky security is measured in shares. At date 4, each share pays a liquidating dividend of one dollar ($\delta = 1$) with probability 0.5 or becomes worthless without paying any dividend ($\delta = 0$) with probability 0.5.

B. **Players.** There are 3 types of players. There are infinitely many atomistic uninformed investors who have no private information about dividend δ. Some uninformed investors, representing one group of investors, experience liquidity shocks and must trade for liquidity reasons. They cannot condition their trades on their beliefs, prices, or other available information. The second group of investors includes all the other uninformed investors who trade at their discretion. The third group consists of an informed investor endowed with W_I dollars who observes a private signal about the dividend δ. Each uninformed investor is endowed with an infinitesimal amount of dollars and shares. However, the aggregate endowment of uninformed investors in both dollars and shares is large relative to the wealth of the informed investor and also relative to the trade size of liquidity investors. All investors consume at date 4 and are risk neutral with respect to their consumption.

C. **Public Signal.** At date 0, everyone observes a public signal s about dividend δ. The binary signal takes a value of 0 or 1 with probability 0.5 each. Conditional on signal s, the probability that $\delta = 1$ is $V(s)$ with

$$V(1) = \theta, \quad V(0) = 1 - \theta. \tag{1}$$

where $0.5 \leq \theta \leq 1$.

D. **Overconfidence.** All investors, whether uninformed or informed, are overconfident. Overconfidence is defined as overestimation of the precision of signal s. Overconfident investors misestimate parameter θ to be $\theta^o > \theta$. They use their biased beliefs to revise the probability that $\delta = 1$ from 0.5 to $V^o(s)$ given by

$$V^o(1) = \theta^o, \quad V^o(0) = 1 - \theta^o \tag{2}$$

We shall use the superscript ^{o} to indicate quantities that are calculated by the overconfident investors based on their irrational beliefs.

E. **Private Signal.** The informed investor observes a private signal r about δ at date 2. The signal takes a value of 0 or 1 with probability 0.5, each independent of the signal s. The realization $r = 1$ raises the probability that $\delta = 1$ by q and the realization $r = 0$ lowers the probability by q where $q \geq 0$ is the quality of the signal r. For now, we assume that the quality q is known. We shall later consider how investors form beliefs about q. Let $V^o(s, r)$ denote the probability that $\delta = 1$ according to overconfident beliefs conditional on signals s and r.

$$\begin{aligned} V^o(1,1) = \theta^o + q, \quad V^o(1,0) = \theta^o - q, \\ V^o(0,1) = 1 - \theta^o + q, \quad V^o(0,0) = 1 - \theta^o - q. \end{aligned} \tag{3}$$

F. **Liquidity Shocks.** The number of investors who face liquidity shocks and the size of these shocks are stochastic. The aggregate liquidity induced dollar demand (supply if negative) for shares, u, follows the distribution function f. We assume that u is integrable[8], zero mean and that the function f is log-concave[9]. We shall see later that log-concavity ensures that share price is a well-behaved (decreasing) function of net demand for shares. The distribution f is common knowledge but no investor observes

[8]A random variable u is integrable if $u^+ \equiv \max(u,0)$ and $u^- \equiv \min(u,0)$ both have finite mean.

[9]Examples of log-concave functions include uniform, (truncated) exponential, and (truncated) normal distribution functions.

the realization of u.

G. **Trading Mechanism.** At date 3, liquidity investors submit their trade u while the informed investor submits its trade x. The uninformed investors collectively act as the market maker. They observe the aggregate order $u+x$ but not u or x separately, use this information to revise their belief about δ, set a competitive price and clear the market.

An order submitted by an investor specifies the dollar amount of shares to buy or sell. I assume that the consumption of each agent must be non-negative at date 4. This is a form of limited liability on the investors and a consequence of this is that their trading strategies are constrained by wealth. Since a share can become worthless in future if it pays no dividend, an investor can place an order to buy shares only up to his wealth in dollars. I rule out short selling to keep analysis simple.[10] Thus, the informed investor with W_I dollars can place order to buy shares for any amount between 0 and W_I dollars. The wealth requirement for trading limits any individual investor's trading choices but not the aggregate trading of the uninformed investors because these investors compete and collectively hold large wealth. I assume that the informed investor does not hold any share before the trading at date 3 so he cannot sell shares at date 3.[11] The assumption is realistic if the informed investor starts only with dollars and others are not sure when the informed acquires information; any buying by the informed investor will be considered to be information motivated and therefore, it will be costly for him to maintain a portfolio of shares and dollars. Figure 1 outlines the sequence of events. The events at date 1 shall be described later.

[10] If short sales limited by the wealth available for trading are allowed, informed investor must choose the amount of shares to buy when the signal is good and also the amount of shares to short sell when the signal is bad. This increases the dimensionality of the strategy space and makes determination of equilibrium characteristics difficult. The intuition for the results, however, should not change with limited short sales.

[11] This assumption is *not* without loss of generality because an informed investor with an optimal portfolio of shares and dollars may be able to sell or buy shares based on his information. The assumption is made for tractability and I believe relaxing the assumption will not change my results qualitatively.

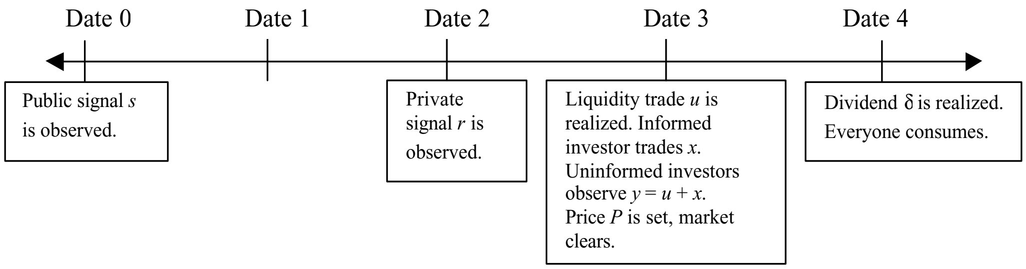

FIGURE 1. Sequence of Events in the Trading Game

H. **Trading Equilibrium with Informed Trading.** Consider the game consisting of events between dates 2 and 4. W_I and s are common knowledge for this game. The liquidity investors cannot choose their trade so we consider only the strategies of the uninformed investors and the informed investor. The informed investor's strategy $X_I(r, s, W_I)$ specifies his trade as a function of his private signal r. The uninformed investors' strategy $P(y, s, W_I)$ specifies the price schedule in terms of the observed aggregate order flow, $y = u + x$. All the quantities in the remainder of this section are calculated using overconfident beliefs as all the players are overconfident. The objective of the informed investor is to maximize his expected trading profit given by

$$\pi_I(W_I) = E_r^o\left[X_I(r, s, W_I) \cdot E_u\left[\frac{V^o(s, r)}{P(X_I(r, s, W_I) + u, s, W_I)} - 1\right]\right]. \quad (4)$$

The expected profit π_I is obtained by taking expectation over signal r (with the probability distribution used by overconfident investors) of the expected profit conditional on r. For each value of r, the informed investor's expected profit equals the product of his dollar demand and the expected profit per unit dollar. The profit per unit dollar equals expected value of dividend (V^o) times the number of shares purchased ($1/P$) minus the investment of one dollar. The uninformed investors compete with each other so they set a price such that their expected trading profit is zero. Their expected trading profit π_U as a function of the order flow y is given by

$$\pi_U(y) = \left(\frac{-y}{P(y, s, W_I)}\right) \cdot \left\{E^o\left[V^o(s, r)\,|y\right] - P(y, s, W_I)\right\}. \quad (5)$$

The expected profit is the product of the number of shares bought and the expected profit per share. The expected dividend is $V^o(s, r)$ but since the uninformed investors do not observe r, they find the expected value of $V^o(s,r)$ conditional on aggregate order flow y. Subtracting the price from this value yields the expected profit per share.

Definition 1. An equilibrium ξ_1 consists of the informed investor's demand function $X_I(r,s,W_I)$, and a price function $P(y,s,W_I)$ such that:

1. $X_I(r,s,W_I)$ maximizes π_I in (4) subject to the constraints $0 \leq X_I(r,s,W_I) \leq W_I$ and
2. $P(y,s,W_I)$ is such that π_U in (5) equals zero for all y.

Equilibrium ξ_1 is a Bayesian equilibrium for the trading game. Condition 1 requires that the informed investor trade to maximize his expected trading profit. Condition 2 represents competition among the uninformed investors.

Lemma 1. *An equilibrium ξ_1 exists for each wealth level W_I of the informed investor.*

This game meets the standard requirements for existence of mixed strategy equilibrium: strategy spaces are nonempty compact subsets of a metric space and payoff functions are continuous. The uninformed investors play a pure strategy in equilibrium because the competitive equilibrium price is unique. Lemma 2 briefly characterizes equilibrium.

Lemma 2. *The informed investor buys shares when $r=1$, does not trade when $r=0$, and makes positive expected profit π_I^*. The price is an increasing function of the aggregate order flow and lies between $V^o(s,0)$ and $V^o(s,1)$.*

The expected value of dividend δ is $V^o(s,\, r)$. The uninformed investors do not observe r but assign positive probabilities to the cases $r=0$ and $r=1$, so they set the price at a value between $V^o(s,\,0)$ and $V^o(s,\,1)$. When $r=0$, shares are overvalued but the informed investor cannot sell short so he doesn't trade. The shares are undervalued when $r=1$, so the informed investor buys shares in this case. We shall refer to the amount bought by the informed investor when $r=1$ as the informed investor's trading strategy. The price is increasing in the order flow because a high order flow indicates a greater probability that the informed investor bought shares and that $r=1$, which increases the probability of high dividend.

If there are multiple ξ_1 equilibria for a wealth level W_I, I assume that the one which maximizes the informed investor's expected trading profit is selected and I refer to this equilibrium as 'the informed trading equilibrium with wealth level W_I.' The notation $X_I^*(W_I)$, $P^*(W_I)$, and $\pi_I^*(W_I)$ will to refer to the informed investor's trading strategy, the uninformed investors' pricing strategy and the informed investor's expected trading profit, respectively in the informed trading equilibrium with wealth W_I.

I. **Price without Informed Trading.** We have seen that informed trading results in the equilibrium price noisily reflecting the informed investor's information. However, it does not eliminate the mispricing arising due to the overconfidence of investors because all the investors are equally overconfident. To isolate the effect of overconfidence on prices, we now show how the price is set when there is no informed trading. In this subsection, we assume there is no information asymmetry and the only players are the uninformed investors and the liquidity investors. In this setting, trade occurs only due to liquidity reasons and the order flow is uninformative about share valuation. Competing uninformed investors set price equal to the expected value of dividend δ and this expectation is independent of the order flow.

Proposition 1. *In the absence of an informed investor, the equilibrium price overreacts to the public signal s. Shares are undervalued when s equals 0 and overvalued when s equals 1. The expected return is negatively correlated with the public signal s as well as with the price.*

The intuition for the result is as follows. A high value of the public signal ($s = 1$) increases the probability that the dividend $\delta = 1$ from 0.5 to $V(1)$. Overconfident uninformed investors overreact to the signal and overestimate this probability to be $V^o(1) > V(1)$. The share price equals the probability that $\delta = 1$ so the shares are priced at $V^o(1)$, overvalued relative to rational price of $V(1)$. Similarly, a low value of the public signal ($s = 0$) lowers the probability that $\delta = 1$ from 0.5 to $V(0)$ but overconfident investors underestimate this probability to be $V^o(0) < V(0)$. The shares are priced at $V^o(0)$ and are undervalued relative to the rational price of $V(0)$. The expected return $V(1)/V^o(1)$ is less than unity when the public signal is high and the price is high while the expected return $V(0)/V^o(0)$ exceeds unity when the public signal is low and the price is low. Thus, the market return on shares from date 3 to date 4 is predictable based on the public signal.

3. RATIONAL AND IRRATIONAL INVESTORS

We have seen that when all investors are irrational, their irrationality impacts price and causes predictability in return. The assumption that all investors are irrational seems unrealistic. This raises the interesting question that what happens when there are some rational investors along with irrational investors. How much can rational investors profit from the arbitrage opportunities that arise due to the effect of irrationality on prices? How do irrational investors respond to the trading strategy of the rational investors? Does the interaction of rational and irrational investors eliminate the effect of irrational investors on the price? To analyze these issues, I consider the interaction between rational and irrational agents. I consider the extreme case in which the aggregate trading wealth of rational investors is small compared to the aggregate trading wealth of the irrational investors. The large wealth of irrational investors is required for them to have aggregate price effect while the assumption that rational investors have small aggregate wealth keeps the analysis tractable by ensuring that the irrational investors are the price setters.

Suppose a rational investor is present along with the uninformed investors and the liquidity investors. The rational investor replaces the informed investor of the previous section so the three types of players now are: infinitely many uninformed investors, liquidity investors whose measure is stochastic, and a rational investor. Like uninformed investors, the rational investor has no access to a private signal. The difference is that while the uninformed investors are overconfident, the rational investor correctly believes that the precision of the public signal s is θ. The rational investor is wealth constrained and his endowment of W_R dollars is negligible relative to the aggregate wealth of the uninformed investors. The rational investor knows that he is rational and that the uninformed investors are irrational. The uninformed investors think they are rational and they are either not aware of the fact that the rational investor has different beliefs or if they are aware, they agree to disagree.

At date 3, liquidity investors and the rational investor submit their trades. Uninformed investors observe the aggregate order flow and set a market-clearing price. They set price equal to the expected value of the dividend according to their beliefs. As we saw earlier, this price V^o represents an overreaction to the signal s. The rational investor can exploit the mispricing, but his trading strategy and his expected profit is constrained by his wealth. Thus, we get the following result:

Proposition 2. *If the rational investor's wealth is small compared to the aggregate wealth of the uninformed investors, the rational investor does not mitigate mispricing due to the trading of irrational agents and his expected trading profit is limited by his wealth.*

The rational investor exploits the predictability in price movement by buying when $s = 0$ and not trading when $s = 1$. His trading strategy is "contrarian" in response to the price overreaction. Any information about his trades does not change the uninformed investors' valuation of shares because they do not believe that the rational investor has superior information. Further, the price equals the uninformed investors' valuation because they compete with each other and their aggregate wealth is large relative to the order flow. Consequently, order flow has no price impact and the rational investor takes as large a position as he can with his wealth. Thus the rational investor's wealth limits his ability to exploit predictability in return.

This result holds even if there are multiple rational investors as long as their aggregate wealth is small so that they do not impact prices. However, if their aggregate wealth is comparable to the aggregate wealth of the uninformed investors, the price is determined jointly by the uninformed investors and the rational investors. In this case, an increase in the wealth of the rational investors increases the size of their trades but also reduces the expected profit per dollar invested.

Now, I ask if the rational investor can borrow money and relax the wealth constraint that limits his expected trading profit. I shall consider both debt and equity contracts. First, consider the case in which the rational investor issues debt for an amount W_B. Suppose the rational investor buys shares for amount X out of the total wealth available $W_B + W_R$. I first assume that the lenders can anticipate X and set a face value of debt $B(X)$. Figure 2 graphically illustrates the debt contracts $B(X)$. The horizontal axis plots the amount that the rational investor invests to buy shares. Line N plots the terminal wealth if shares do not pay any dividend. It equals the original wealth $W_B + W_R$ minus investment of X to buy X/V^o shares (as the uninformed investors price shares at V^o). Line M plots the terminal wealth if the shares pay dividend $\delta = 1$, that exceeds the wealth in N by the dividend X/V^o. If $X < W_R$, the rational investor does not use borrowed funds for trading, the debt is riskless and $B(X) = W_B$. If $X > W_R$, debt is risky and the rational investor gets a positive payoff after repayment only in the high state ($\delta = 1$). Since the uninformed investors believe this happens with probability V^o, they set face value so that the rational investor is left with fixed amount W_R/V^o. Thus, $B(X)$ is a line parallel to M at a vertical distance W_R/V^o.

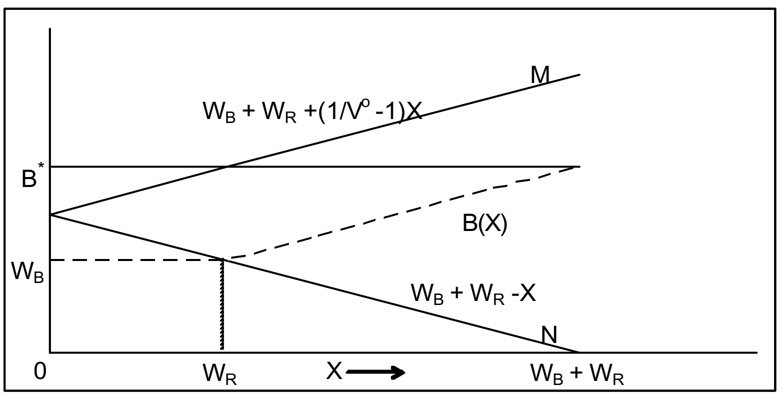

FIGURE 2. Payoffs and Pricing of Debt Contracts

Next, I relax the assumption that lenders can anticipate X. Since they cannot control the rational investor's trade, they expect him to trade so as to minimize the market value of debt. Anticipating the worst outcome, lenders choose the highest face value, which is B^* in Figure 2. With this face value, the rational investor's net payoff in the high state is the distance between lines M and B^*. He chooses to trade with all wealth, gets nothing in the low state, and gets W_R/V^o in the high state with probability V. His maximum expected profit from borrowing is $W_R(V/V^o - 1)$ which is the expected profit he would have made by trading with personal wealth.

If the rational investor issues equity, his expected profit does not change at all. The reason is that the equityholders get the same share of profit as their share in the total wealth invested and the return is independent of the level of investment, so the rational investor's residual profit exactly equals the profit that he makes by investing his personal wealth. Thus, we have proved the following result.

Proposition 3. *The rational investor cannot increase his expected trading profit by raising external finance through debt or equity.*

The intuition for Proposition 3 is that uninformed investors undervalue the debt or equity stake in the rational investor's profit. In other words, they charge too much for giving him external finance because they underestimate his profit. When the rational investor issues equity, equityholders share in his profit. The rational investor's trade can be decomposed into a trade with his personal wealth and an identical trade with the issued equity, and the equityholders get the proceeds from the latter trade so the rational investor's profit is limited to what he earns trading solely with his own wealth. In case of debt, the amount borrowed can be decomposed into a riskless part and a risky part. The risky part is equivalent to equity when there are only two states of payoffs ($\delta = 0$ or $\delta = 1$), so it does not increase the rational investor's profit. The riskless part is the part of wealth that the rational investor does not trade, so it too does not increase his expected trading profit.

4. FINANCIAL INTERMEDIATION

The previous section shows that the rational investor's expected profit is limited by his personal wealth. Raising external financing does not increase his expected profit because uninformed investors do not concede that the rational investor has a trading advantage arising from their irrationality; consequently, the terms at which they provide financing are unfavorable to the rational investor. The rational investor can overcome this problem if the uninformed investors are convinced that he has a trading advantage, even if they do not concede that they are irrational. In this case, the rational investor can raise external financing from uninformed investors for trading at terms which provide him an expected profit greater than what he can earn trading solely on his own account. In other words, the rational investor can form a financial intermediary in which uninformed investors invest because they believe that the intermediary has a trading advantage over them.

I show that if the uninformed investors assign a non-zero probability to the possibility of an investor having access to a private signal, they will eventually believe that the rational investor has access to a private signal and that he has a trading advantage arising from the superior information of his private signal. This will enable the rational investor to form an intermediary with expected profit exceeding that possible with his personal wealth.[12] I show this in two steps. In Subsection A I analyze the steady-state case in which the uninformed investors believe they know the quality of information available to the intermediary precisely, while in Subsection B I examine a setting in which the uninformed investors learn and update their beliefs about the intermediary over a number of periods to reach the steady-state beliefs of Section A. Finally, in Subsection C, I discuss the consistency of the uninformed investors' beliefs.

A. **The Steady State Case: Irrational Agents Believe there is an Informed Agent.** The setup is similar to that in Section 3 except that the rational investor now operates a financial intermediary and the uninformed investors believe that the intermediary has access to private information about dividend δ. In particular, they believe that at date 2, the intermediary observes signal r. Thus, the uninformed investors misperceive the rational investor as the informed investor defined in Section 2. Formally, the uninformed investors believe

$\mathcal{A}$.: The intermediary is operated by the informed investor.

$\mathcal{B}$.: $\mathcal{A}$ is common knowledge.

[12] I do not explore the option of directly selling information to some uninformed investors. Admati and Pfleiderer (1988) study the decision to sell or to trade on information in financial markets while Admati and Pfleiderer (1990) compare direct sale of information to indirect selling by forming a mutual fund. Allen (1990) shows that when risk-aversion is unobservable and assessing a claim of superior information is difficult, a seller of information can capture greater value of his information by forming a financial intermediary. I do not attempt to determine the optimal mechanism to exploit the trading advantage. The problem is complicated in my model because of diverse beliefs of the rational and irrational investors about the source of the advantage of the intermediary.

I assume that the uninformed investors' estimate of the quality q of the intermediary's supposed signal r is a number not exceeding $q^* \equiv \theta^o - \theta$. Subsection B shows that this is indeed the case when the uninformed investors' beliefs are an outcome of learning. To show that financial intermediation helps the rational investor earn positive expected profit that is not limited by his personal wealth, I determine equilibrium for the interaction of the intermediary and the other investors when the rational investor has already formed a financial intermediary. Any systems necessary for intermediation have been set up and the associated cost already incurred.

A.1. *Nature of Financial Intermediary.* A financial intermediary offers to invest on behalf of other investors for a fee, collects from investors the amount they wish to invest through the intermediary as well as fees, trades with the invested money and returns trading proceeds to the investors. Examples of such intermediaries are mutual funds or hedge funds, in which, one or more fund managers actively manage portfolios. This is in contrast to index funds whose investment strategy is passive or brokers who execute the trades that their clients want.

A.2. *Sequence of Events.* The public signal s is observed at date 0 and at date 1, the intermediary operated by the rational investor announces the terms for raising funds. It announces the aggregate investment F sought from the uninformed investors and also the fraction k of invested funds that will be charged as a fee. The intermediary precommits to return any funds collected beyond F. Each uninformed investor determines how much to invest and pays the corresponding fee to the intermediary. If aggregate funds exceed F, the intermediary returns the surplus funds and the associated fee to the investors.

At date 3, liquidity investors and the intermediary submit their orders. The intermediary can use only the invested funds for trading. I assume there are mechanisms in place that preclude conflicts of interest i.e., prevent the intermediary from trading on its own account against the interest of the investors it has raised money from. The aggregate order flow is observed by the uninformed investors who set a market-clearing price. At date 4, the dividend δ on shares is realized and the trading profit of the intermediary is determined. The total fee collected by the intermediary is called its revenue. In contrast, the intermediary's trading profit is the profit realized from the intermediary's trading using the invested funds. With a fixed fee the revenue of the intermediary is independent of the realized trading profit. The intermediary returns the total proceeds to the investors at date 4. I assume that the total proceeds are verifiable (for example, with audits), but investors cannot use this information to infer the intermediary's exact trade at date 3. They can however, infer whether or not the intermediary bought shares. This coarse information indicates the trading position of the intermediary without revealing the trading strategy. With this assumption, the uninformed investors cannot discover a discrepancy between the actual trade of the intermediary and the possible trades that they would expect if the intermediary were operated by the informed investor. The justification for the assumption is the real-life difficulty of inferring the precise trading strategy of financial intermediaries just by observing their annual performance. I implement this assumption by specifying the intermediary's total proceeds as the proceeds from trading plus a zero-mean random term; only this sum is verifiable and returned to the investors. These events are repeated every period. The sequence of events is outlined in Figure 3.

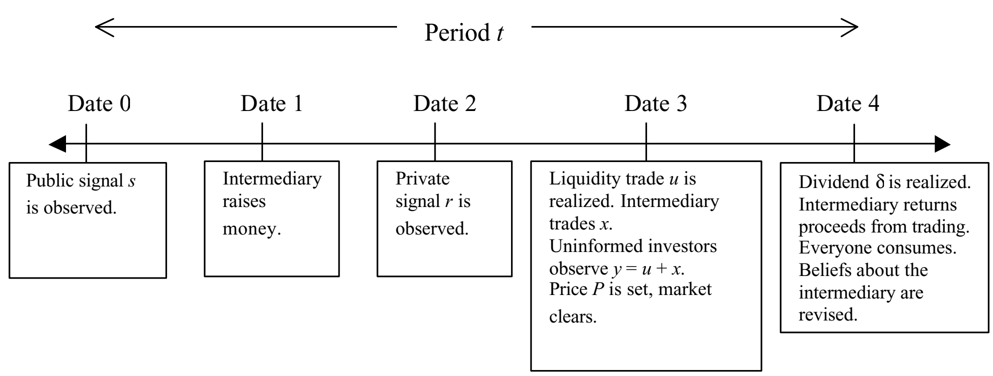

FIGURE 3. Sequence of Events with Financial Intermediation

A.3. *Equilibrium.* I define below the equilibrium for the events from dates 2 to 4. The public signal s observed before this game affects the equilibrium. For notational simplicity, I suppress the argument s in some of the following expressions.

Definition 2. An equilibrium ξ_2 consists of:

1. The uninformed investors' belief about the intermediary's trading strategy $X_I(r, s, D)$

such that $0 \leq X_I(r, s, D) \leq D$,

2. The uninformed investors' pricing function $P(y, s, D)$ such that π_U in (5) equals zero for all y, given strategy X_I,
3. The uninformed investors' investment function $D(F, k) \leq F$ such that

$$\pi_I(D) \begin{cases} \leq kD & \text{if } D = 0, \\ = kD & \text{if } 0 < D < F, \\ \geq kD & \text{if } D = F, \end{cases}$$

given strategies X_I and P,

4. The intermediary's offer (F, k) that maximizes $kD(F, k)$ given strategy D,
5. The intermediary's trading strategy $X_R(s, D)$, $0 \leq X_R(s, D) \leq D$.

This game has prior beliefs of the rational investor and the uninformed investors that are inconsistent with each other. The uninformed investors believe that they are playing against the informed investor and that this is common knowledge. However, the rational investor understands the structure of the game. The ingredients of the equilibrium include the intermediary's terms: the fund size F and the fee k, the uninformed investors' aggregate investment strategy D, their belief about the intermediary's trading strategy, X_I, their price setting strategy P, and the intermediary's true trading strategy X_R. All agents maximize consumption in current and future periods. Since no learning is involved, events in a given time period do not influence future events so the agents simply maximize current period's consumption.

The equilibrium satisfies all the properties of a Bayesian Nash equilibrium except for common priors. Not only do irrational investors not know that they are playing against the rational investor and not the informed investor, but they also do not realize that if they were irrational then the rational investor would appear to be the informed investor to them. They assign zero probability to this possibility. In other words, they do not know what they do not know.[13] Note that this kind of belief structure follows simply from the specification that the uninformed investors process information incorrectly but they do not suspect so (otherwise they will learn to be rational over time). Each investor

[13] See Geneakoplos (1994) and Morris (1995) for discussion of related issues.

is rational in its actions and belief revision conditional on his prior beliefs.

Condition 1 simply says that the uninformed investors have feasible belief about the intermediary's trading strategy. An explicit incentive compatibility constraint for the intermediary to follow this trading strategy is not needed because it can not gain by choosing an alternative trading strategy; its fee is independent of the trading outcome. Condition 2 ensures that the uninformed investors set price competitively given their belief $\mathcal{A}$ and their belief that the intermediary trades with strategy X_I. Condition 3 represents competition and individual rationality of the uninformed investors in their investment decision. When they invest D in the intermediary, they pay a fee kD and expect to receive trading profit of π_I. They are either indifferent towards a marginal investment in the intermediary ($kD = \pi_I$) or there is a corner solution with zero or maximum investment. Condition 4 represents the intermediary's revenue maximization. Condition 5 does not restrict the intermediary's trading strategy X_R except for wealth requirements. The intermediary's revenue is unaffected by the trading outcome, so it is unconcerned about which trading strategy it chooses, while the uninformed investors cannot observe the chosen trading strategy.

Proposition 4. *The following outcome is supported by an ξ_2 equilibrium:*

1. *The size of funds sought by the intermediary is the minimum wealth level that maximizes expected trading profit of the informed investor;* $F = \inf \underset{W_I}{argmax}\, \pi_I^*(W_I)$,
2. *The intermediary seeks revenue equal to the maximum expected trading profit of the informed investor;* $k = \pi_I^*(F)/F$,
3. *The uninformed investors invest the funds sought by the intermediary,* $D = F$,
4. *If* $r = 0$, $X_I = 0$ *and if* $r = 1$, $X_I = F$,
5. *The uninformed investors' price schedule is*

$$P(y) = \frac{V^o(s,1) f(y-F) + V^o(s,0) f(y)}{f(y-F) + f(y)},$$

6. *If* $s = 0$, $X_R = F$ *and if* $s = 1$, $X_R = 0$.

Proposition 4 shows that in equilibrium, the intermediary raises the funds that the hypothetical informed investor would require to maximize its expected trading profit. The reason is that the intermediary can earn only as much revenue as the uninformed investors expect it to earn in trading profit. Since they believe that the intermediary is operated by the informed investor, the intermediary raises funds that the uninformed investors think will enable the informed investor to maximize expected trading profit. It then sets a fee so as to capture all the expected trading profit that the uninformed investors expect it to earn. The uninformed investors provide funds because they believe that the revenue equals expected trading profit, and they are competitive. The intermediary can use any arbitrary trading strategy because there is no learning and the uninformed investors do not base future decisions on the past performance of the intermediary. However, the proposed equilibrium trading strategy X_R is the one that maximizes the expected trading profit of the intermediary. Clearly, $X_R = 0$ when $s = 1$, because the shares are overvalued and the intermediary cannot short sell. When $s = 0$, the shares are undervalued and the intermediary buys using the entire funds F. To see why this strategy maximizes expected profit, consider the trading strategy that uninformed investors expect the hypothetical informed investor to follow. They expect the informed investor to maximize expected profit by buying F when it observes $r = 1$ and has valuation $V^o(0, 1) = 1 - \theta^o + q$. Since the intermediary's valuation $V(0) = 1 - \theta$ is even higher (as $q \leq \theta^o - \theta$), it must buy at least F to maximize profit. Since it cannot buy more than F, it buys exactly F.

Proposition 4 shows that the size of the funds raised equals the minimum wealth that enables the hypothetical informed investor to maximize expected trading profit. It may not be obvious that there is a finite wealth level beyond which additional wealth does not increase the informed investor's expected trading profit. The following lemma shows that this is the case.

Lemma 3. *The informed investor's equilibrium expected trading profit attains a maximum for a finite* W_I*.*

The intuition is that the informed investor acts as a monopolist and trades off the better exploitation of mispricing possible by increasing his trade size against the reduction in mispricing due to the information revealed by his trade. When he assumes a very large trading position, it becomes difficult to hide his trade among the trades of liquidity investors. As trades become more transparent, mispricing decreases and the informed investor's expected profit diminishes. Thus, the intermediary's expected profit is maximized at a finite level of wealth invested in the market. However, this profit is not limited by the rational investor's personal wealth so intermediation may allow a rational investor with limited wealth to increase expected profit.

A.4. *Trading Performance of the Uninformed Investors.* Do the incorrect beliefs of the uninformed investors help them or hurt them in their investment decision in the intermediary and in their trading at date 3? I first discuss their trading performance, i.e., whether they make average profit or loss when they clear market at date 3. Recall that they are competitive and believe that they are making zero expected profit in setting a market clearing price.

Proposition 5. *The uninformed investors make negative expected trading profit when the public signal s is 0 and shares are undervalued. They make positive expected trading profit at the expense of liquidity investors when the public signal s is 1 and shares are overvalued.*

The intuition behind Proposition 5 is as follows. When $s = 0$, shares are undervalued and the intermediary always buys. The uninformed investors think that the intermediary buys only half the time (when $r = 1$) so they underestimate the information content of order flow and lose money due to adverse selection. When $s = 1$, shares are overvalued, and the intermediary does not trade. The uninformed investors still expect the intermediary to buy with probability 0.5. They wrongly believe that order flow is informative and move prices in response to the order flow of liquidity investors. This hurts liquidity investors who lose money on average to the uninformed investors.

A.5. *Contracts and Financial Intermediary's Performance.* I have so far assumed that the intermediary's revenue is independent of performance. When there is no learning, the uninformed investors believe there is no reason for the intermediary to not maximize expected trading profit. Thus, they are indifferent to paying a fixed fee to the intermediary or making the fee contingent on the trading profit. However, since the intermediary has different beliefs, it is not clear whether it prefers a fixed fee or a performance-contingent fee. I focus on contracts that pay the intermediary a fixed fee and a fixed fraction of the trading profit.[14] Competition among the uninformed investors results in contracts that pay a fixed fee of $\alpha\pi_I^*(D)$ and a fraction (1-α) of the realized profit where α is a positive constant. The wealth of the intermediary places an upper bound on α because it cannot have negative consumption. The uninformed investors are indifferent between contracts with different values of α. However, the intermediary prefers higher or lower α depending on how the true expected trading profit π_R compares with π_I^*, the profit that the uninformed investors expect the intermediary to make. Thus, the intermediary's preference for contracts is related to the profitability of investment in the intermediary. When investors in the intermediary receive expected profit that is more (less) than the fee they pay to the intermediary, the intermediary prefers to maximize (minimize) the profit sensitivity of revenue.

Proposition 6. *When $s = 0$, investment in the intermediary is profitable and the intermediary prefers a more-performance-contingent contract over a less-performance-contingent one. When $s = 1$, investment in the intermediary is unprofitable and the intermediary prefers a less-performance-contingent contract over a more-performance-contingent one.*

The intuition is as follows. The uninformed investors believe that the trading advantage of the intermediary is due to superior information rather than their irrationality. Thus, they fail to grasp the correlation between the public signal s and the profit of the intermediary. With no short sales, the correlation between the intermediary's expected trading profit and the signal s is extreme; the intermediary makes higher expected profit than what the uninformed investors expect when $s = 0$ and makes zero profit when $s = 1$. Thus, those who invest in the intermediary pay a fee that is too low when $s = 0$ and pay a fee that is too high when $s = 1$. In response, the intermediary prefers performance-contingent contract in the former case and a fixed fee in the latter case.

Proposition 4 shows that the rational investor can form an intermediary and attract funds if the uninformed investors believe that the intermediary can trade more profitably than they can. This raises two questions: Is the rational investor always better off

[14] I consider linear contracts because they can be ranked unambiguously by their performance sensitivity. The results should hold qualitatively for more general contracts where this ranking is possible.

creating an intermediary rather than trading on his own account? And can anyone else create an intermediary and attract funds? The answers to both questions depend on what determines the beliefs of the uninformed investors about the intermediary's ability to trade profitably. The next subsection shows how these beliefs evolve.

B. **The Transient Case.** Consider a period t in which the uninformed investors are uncertain about the quality q of the intermediary's supposed private signal. Their beliefs about q are represented by a probability distribution. We need $q \leq 1 - \theta^o$ for the probabilities in (3) to be well-defined. Thus, the support of the distribution is [0, 1 - θ^o]. The distribution at the beginning of period t (or end of period $t-1$) is denoted by g_t. The uninformed investors update the distribution at the end of each period using information they get in that period. They use observed dividend and their inference about whether or not the intermediary bought shares to determine if the trading position of the intermediary was profitable in hindsight and to update their beliefs. Two differences from the earlier steady state analysis are: (1) the quality of information q is not known with certainty, and (2) the agents' actions in a period can influence their payoffs in subsequent periods.

Since the uninformed investors incorrectly think that the intermediary is operated by the informed investor, an equilibrium must specify their beliefs about the informed investor's strategy. To do this, we shall first analyze equilibrium for a hypothetical game in which the intermediary is actually operated by the informed investor.

B.1. *Equilibrium for the Hypothetical Case with Informed Intermediary.* Let q_t be the expected value of q based on the distribution g_t. The probability of each of the four possible realizations of (s, r) is 0.25 each and irrational investors estimate the probability V_t^o that $\delta = 1$ conditional on signals (s, r) as

$$\begin{aligned} V_t^o(1,1) &= \theta^o + q_t, \quad V_t^o(1,0) = \theta^o - q_t, \\ V_t^o(0,1) &= 1 - \theta^o + q_t, \quad V_t^o(0,0) = 1 - \theta^o - q_t. \end{aligned} \tag{6}$$

The intermediary is operated by the informed investor and offers at date 1 to invest on behalf of investors at the terms (F_t, k_t). The uninformed investors provide funds D_t and the associated fee k_tD_t. Trading takes place at date 3. The informed investor's trading strategy is to buy shares for $X_{It}(r, s, D_t)$ when the private signal is r, the public signal is s, and the wealth available for trading is D_t. The uninformed investors' pricing strategy sets price $P_t(y, s, D_t)$ when the aggregate order flow is y, the public signal is s, and the wealth available for trading to the intermediary is D_t. The expected trading profit of the intermediary is

$$\pi_{It}(s, D_t) = E_r\left[X_{It}(r, s, D_t) \cdot E_u\left[\frac{V_t^o(s, r)}{P_t(X_{It}(r, s, D_t) + u, s, D_t)} - 1\right]\right], \tag{7}$$

and the expected trading profit of the uninformed investors is

$$\pi_{Ut}(y) = \left(\frac{-y}{P_t(y, s, D_t)}\right) \cdot \left\{E\left[V_t^o(s, r)\,|y\right] - P_t(y, s, D_t)\right\}. \tag{8}$$

As in Section 2, we can define the equilibrium quantities as a function of the trading wealth of the informed investor. An informed trading equilibrium with wealth D_t is defined as a (X_{It}, P_t) pair such that $0 \leq X_{It} \leq D_t$, X_{It} maximizes π_{It} for given P_t, and P_t sets π_{Ut} to zero for given X_{It}. 'The trading equilibrium with wealth D_t' is defined as an informed trading equilibrium with wealth D_t that maximizes π_{It}. We shall use $X_{It}^*(D_t)$, $P_t^*(D_t)$, and $\pi_{It}^*(D_t)$, respectively to refer to the informed investor's trading strategy, the uninformed investors' pricing strategy and the informed investor's expected trading profit in the trading equilibrium with wealth D_t.

The trading outcome in a period affects not only the profit of the investors in the intermediary during that period but also distribution g_{t+1}, which determines the equilibrium in period $t+1$ and hence the intermediary's expected future revenue. The trading strategy that maximizes the intermediary's expected trading profit in the current period may differ from one that maximizes the intermediary's expected future revenue in which case the intermediary will not maximize expected trading profit. The uninformed investors take this into account in determining the intermediary's expected trading profit and the fee they pay to the intermediary. The strategic issues that arise due to the divergence between the objectives of the intermediary and the investors are interesting in their own right but solving this problem is difficult in the current model.[15] Incentive contracts can be adapted to cope with the agency problem of career concerns. With risk-neutrality, the only factor that limits the profit sensitivity of the intermediary's fee is its wealth. I make the following assumptions in order to ignore the effect of career concerns on the intermediary's decisions:

a: The uninformed investors have strong priors about q, so the revision in beliefs is slow and the intraperiod discount factor is small enough so that the impact of the intermediary's performance in one period on the discounted revenue from future periods is small.

b: The intermediary's revenue consists of a fixed fee and a sufficiently large fraction of the trading profit.

Assumptions a and b align the intermediary's incentives with those of the investors so the intermediary maximizes expected trading profit rather than optimize its own future revenue. The intermediary's revenue consists of a fixed fee and an exogenous fraction $\alpha > 0$ of the trading profit. I now define equilibrium for this game.

Definition 3. An equilibrium ξ_3 consists of:

1. The informed investor's trading strategy $X_{It}(r, s, D_t)$ that maximizes π_{It} subject to the constraints $0 \leq X_{It}(r, s, D_t) \leq D_t$, given strategy P_t,
2. The uninformed investors' pricing function $P_t(y, s, D_t)$ such that π_{Ut} equals zero for all y, given strategy X_{It},
3. The uninformed investors' investment function $D_t(F_t, k_t) \leq F_t$ such that

$$(1-\alpha)\,\pi_{It}\,(D_t) \begin{cases} \leq k_t D_t & \text{if } D_t = 0, \\ = k_t D_t & \text{if } 0 < D_t < F_t, \\ \geq k_t D_t & \text{if } D_t = F_t, \end{cases}$$

given strategies X_{It} and P_t,

[15] Holmstrom (1999) and Milbourn, Shockley, and Thakor (2001) analyze how a manager's decision to supply effort and to invest in information production about projects, respectively are influenced by career concerns. Chevalier and Ellison (1997) and Brown, Goetzmann, and Park (2001) show evidence of career concerns and importance of reputational costs in mutual funds and hedge funds, respectively.

4. The intermediary's offer (F_t, k_t) that maximizes $k_t D_t(F_t, k_t) + \alpha \pi_{It}(D_t(F_t, k_t))$ given strategies P_t, X_{It}, and D_t,
5. Bayesian revision from g_t to g_{t+1} using the information available to the uninformed investors in period t.

Condition 1 requires that the intermediary operated by the informed investor trade to maximize expected trading profit; this is the intermediary's incentive compatibility constraint because its revenue is linearly increasing in trading profit. Condition 2 arises from competition among the uninformed investors while setting price. Condition 3 represents competition and the individual rationality of the uninformed investors in their investment decisions. They pay a fee $k_t D_t$ and receive a fraction $1-\alpha$ of expected trading profit π_{It}. They are either indifferent towards a marginal investment in the intermediary ($k_t D_t = (1-\alpha)\pi_{It}$) or there is a corner solution with zero or maximum investment. Condition 4 represents the intermediary's expected revenue maximization. Condition 5 requires that the belief about the quality q of the intermediary's information be updated rationally.

Proposition 7. *The following outcome is supported by an ξ_3 equilibrium:*

1. *The informed investor seeks funds equal to the minimum wealth level that maximizes expected trading profit;* $F_t = \inf \underset{D_t}{argmax}\, \pi^*_{It}(D_t)$,
2. *The informed investor seeks a fixed fee equal to fraction $(1-\alpha)$ of the maximum expected trading profit;* $k_t = (1-\alpha)\pi^*_{It}(F_t)/F_t$,
3. *The uninformed investors invest the funds sought by the intermediary,* $D_t = F_t$,
4. *If $r = 0$, $X_{It} = 0$ and if $r = 1$, $X_{It} = F_t$,*
5. *The uninformed investors' price schedule is*

$$P_t(y) = \frac{V_t^o(s,1)\, f(y - F_t) + V_t^o(s,0)\, f(y)}{f(y - F_t) + f(y)},$$

6. $g_{t+1}(q) = \frac{\psi(q) g_t(q)}{\int \psi(\hat{q}) g_t(\hat{q}) d\hat{q}}$ *where* $\psi(q) = (2\theta^o - 1)\mathbf{1}_{s=\delta} + 2q\mathbf{1}_{b=\delta} + 1 - \theta^o - q$ *and b is 1 if the intermediary buys and 0 otherwise.*

Proposition 7 shows that when the intermediary is operated by the informed investor, it raises just enough funds to maximize its expected trading profit. The intermediary gets a fixed fee and a fraction of the trading profit such that its expected revenue equals the expected trading profit. Thus, the intermediary captures all the rents from its superior information. The uninformed investors provide funds because they expect to recover fee paid to the intermediary from the trading profit they get on their investment. The intermediary uses funds raised to buy shares when its private signal is good ($r = 1$) but does not buy otherwise. At the end of each period, the uninformed investors observe whether the intermediary bought shares or not and use this to infer his private signal r. They use the inferred values of signals s and r and the realized dividend δ and employ Bayes rule to recalibrate their beliefs about the quality q of the intermediary's private signal.

B.2. *Equilibrium with Rational Intermediary.* We now return to the setting in which the intermediary is operated by a rational investor but the uninformed investors believe that it is operated by an informed investor. The uninformed investors are uncertain about the quality q of the intermediary's supposed private signal. From the point of view of the uninformed investors, this situation is identical to that in subsection B.1. However, the intermediary observes no private signal and its expected trading profit with the trading strategy X_{Rt} is given by

$$\pi_{Rt}(s, D_t) = X_{Rt}(s, D_t) \cdot E_u \left[\frac{V_t(s)}{P_t(X_{Rt}(s, D_t) + u, s, D_t)} - 1 \right]. \tag{9}$$

The equilibrium for this case is defined as:

Definition 4. An equilibrium ξ_4 consists of:

1. The uninformed investors' belief about the intermediary's trading strategy $X_{It}(r, s, D_t)$ that maximizes π_{It} such that $0 \leq X_{It}(r, s, D_t) \leq D_t$, given strategy P_t,
2. The uninformed investors' pricing function $P_t(y, s, D_t)$ such that π_{Ut} equals zero for all y, given strategy X_{It},
3. The uninformed investors' investment function $D_t(F_t, k_t) \leq F_t$ such that

$$(1-\alpha)\,\pi_{It}(D_t) \begin{cases} \leq k_t D_t & \text{if } D_t = 0, \\ = k_t D_t & \text{if } 0 < D_t < F_t, \\ \geq k_t D_t & \text{if } D_t = F_t, \end{cases}$$

given strategies X_{It} and P_t,

4. The intermediary's trading strategy $X_{Rt}(s, D_t)$ that maximizes π_{Rt} subject to the constraints $0 \leq X_{Rt}(s, D_t) \leq D_t$, given strategy P_t,
5. The intermediary's offer (F_t, k_t) that maximizes $k_t D_t(F_t, k_t) + \alpha \pi_{Rt}(D_t(F_t, k_t))$ given strategies P_t, X_{Rt}, and D_t,
6. Bayesian revision from g_t and the information obtained by the uninformed investors in period t to function g_{t+1}.

This equilibrium is similar to the equilibrium in Definition 3 where the informed investor operates the intermediary. In fact, the uninformed investors cannot distinguish between the two cases. However, the two conditions governing the equilibrium strategies of the intermediary are different. First, Condition 4 requires that the rational intermediary trade on the basis of the public signal so as to maximize its expected trading profit in (9). Second, in maximizing expected revenue in Condition 5, the intermediary not only takes into account the uninformed investors' expectation of its profit but also the true expected profit which depends on how uninformed investors expect it to trade as well as on its true trading strategy.

Proposition 8. *The following outcome is supported by an ξ_4 equilibrium:*

1. *The intermediary seeks funds equal to the minimum wealth level that maximizes expected trading profit of the informed investor; $F_t = \inf \underset{D_t}{argmax}\, \pi^*_{It}(D_t)$,*
2. *The intermediary seeks a fixed fee equal to fraction $(1-\alpha)$ of the maximum expected trading profit of the informed investor; $k_t = (1-\alpha)\pi^*_{It}(F_t)/F_t$,*
3. *The uninformed investors invest the funds sought by the intermediary, $D_t = F_t$,*
4. *If $r = 0$, $X_{It} = 0$ and if $r = 1$, $X_{It} = F_t$,*
5. *The uninformed investors' price schedule is*

$$P_t(y) = \frac{V_t^o(s,1) f(y - F_t) + V_t^o(s,0) f(y)}{f(y - F_t) + f(y)},$$

6. *If $s = 0$, $X_{Rt} = F_t$ and if $s = 1$, $X_{Rt} = 0$,*
7. $g_{t+1}(q) = \frac{\psi(q) g_t(q)}{\int \psi(\hat{q}) g_t(\hat{q}) d\hat{q}}$ *where $\psi(q) = (2\theta^o - 1)\mathbf{1}_{s=\delta} + 2q\mathbf{1}_{b=\delta} + 1 - \theta^o - q$ and b is 1 if the intermediary buys and 0 otherwise.*

This proposition shows that the financial intermediary operated by the rational investor attracts funds and earns positive revenue when the uninformed investors think that the intermediary is operated by the informed investor and when the intermediary's revenue is made sensitive to profit. Proposition 8 is similar to Proposition 4 for the steady state case except that the revenue of the intermediary is contingent on the trading profit and the uninformed investors are learning.

The expected revenue of the intermediary equals $(1-\alpha)\,\pi^*_{It}(F_t)$ from its fixed fee plus $\alpha\pi^*_{Rt}(F_t)$ from its trading profit share. The true expected profit (π^*_{Rt}) differs from what the uninformed investors expect (π^*_{It}). When $s=0$, shares are undervalued at the equilibrium price and the intermediary buys to exploit mispricing while the uninformed investors expect it to buy only when its supposed private signal $r=1$, which occurs with probability 0.5. Thus, the true expected trading profit of the intermediary is twice what the uninformed investors expect. When $s=1$, shares are overvalued at the equilibrium price, but the intermediary cannot sell short, so its expected trading profit is zero. Consequently, the intermediary expects to obtain a total revenue of $(1+\alpha)\,\pi^*_{It}(F_t)$ or $(1-\alpha)\,\pi^*_{It}(F_t)$, depending on the realization of s. In either case, the intermediary can maximize its expected revenue by maximizing the expected profit π^*_{It} perceived by the intermediaries. Thus, the intermediary chooses the same terms for raising funds as an intermediary operated by the informed investor would. It demands a fee that the uninformed investors consider fair, so that these investors are indifferent to a marginal investment in the intermediary. Finally, the belief revision of the uninformed investors employs Bayes rule but is based on the incorrect assumption that the intermediary buys if and only if $r=1$. However, it gives them a measure of the trading advantage of the intermediary as it measures how often the intermediary's contrarian trading strategy is successful ($b=\delta$).

Proposition 9. *The uninformed investors' estimate of the intermediary's private information, q_t, converges to $q^* \equiv \theta^o - \theta$.*

Proposition 9 shows that if the intermediary is operated by the rational investor, the uninformed investors will gradually revise their estimate of quality of the intermediary's information to q^*. The quantity q^* equals $\theta^o - \theta$, and is a measure of the irrationality of the uninformed investors. The learning rule captures the extent to which the intermediary's trades predict δ more accurately than the prediction based on uninformed investors' biased interpretation of the public signal. Since the intermediary does not truly possess superior information, the bias in the uninformed investors' beliefs $\theta^o - \theta$ is interpreted as the quality of the intermediary's information. Thus, the greater the irrationality of the uninformed investors, the greater is the steady-state quality of the intermediary as perceived by the uninformed investors. In a more general model with a cost of formation of intermediary, the incentives for formation of an intermediary by the rational investor will depend on the cost of operating the intermediary, the expected revenue of the intermediary and the expected profit if the rational investor trades with only his personal wealth. If the uninformed investors are only slightly irrational, q^* may not be large enough to justify the cost of operating an intermediary. The larger the personal wealth of the rational investor, the less is the benefit of raising funds from investors to trade and thus the lower is the attractiveness of intermediation.

A related issue is whether any uninformed investor can form a financial intermediary even if it has no trading advantage. I do not consider the issue of what happens if uninformed investors form intermediary. In a more general model, anyone will be able to form a financial intermediary, but it is the uninformed investors who will determine the profitability of the intermediary. An intermediary has to demonstrate by performance that it can trade more profitably than others and thus build a track record. If the uninformed investors start with a low estimate of intermediary's quality, an intermediary with no trading advantage will not perform better than the uninformed investors on average. Eventually, its revenue will be too low to justify the cost of operating the intermediary.

The following result shows how intermediation affects mispricing.

Lemma 4. *The average mispricing decreases as the size of funds raised by the intermediary increases.*

Financial intermediation helps the rational investor exploit the predictability in prices due to the irrationality of the uninformed investors. As the size of the intermediary's trade increases, the average mispricing decreases because the intermediary's trading position is more easily inferred from the order flow. However, mispricing does not vanish because the intermediary does not want to “overfish the pond” by taking too large a trading position. Thus, the intermediary acts strategically and limits the size of its trade so as not to reveal too much information in the order flow and reduce its profit.

C. **Consistency of Beliefs.** I have shown that when the intermediary is rational and the uninformed investors start with a non-zero prior belief that the financial intermediary's information is superior, then over time their estimate of the quality of the intermediary's information increases. A natural question is why I consider non-zero priors of the uninformed investors about the superior information of intermediary, as opposed to a non-zero prior belief about their own irrationality. This is related to the question: how irrational are the uninformed investors? The uninformed investors are overconfident about the public signal s, but they are completely rational in all other aspects and follow Bayes rule in updating their beliefs. The learning process critically depends on prior beliefs. If these investors assign positive probability to the fact that they are overconfident, repeated observations of their performance will eventually cause them to learn that they are irrational, they will thus become rational.[16] I assume that irrational investors continue to be irrational for a long time, thereby ruling out any mechanism that eliminates irrationality.

The assumption of a non-zero prior belief about the possibility of informationally-advantaged investors is innocuous. It seems reasonable that when irrational investors view the rational investor repeatedly earning a higher profit than they do, they will seek an explanation. The opposite assumption that irrational investors simply ignore past evidence seems implausible. I have specified this alternative explanation to be an informational advantage. However, it could be any other advantage, as long as that advantage has lasting power and yields irrational investors a higher trading profit than

[16]It is possible that irrationalities are like addiction or otherwise genetically hard-wired, that is, these behavioral anomalies cannot be controlled even if you know about them. It is common for people to impose rigid constraints on themselves like budgeting, new year promises which would be unnecessary if people were completely rational all the time or if they were unaware of their irrationality. This view of irrationalities would imply that irrational investors may recognize their own irrationality and yet invest with rational intermediaries as a precommitment device not to act irrationally with their money.

they can earn on their own.

5. EXTENSION AND EMPIRICAL IMPLICATIONS

Subsection A considers an extension to multiple rational investors and Subsection B draws empirical predictions.

A. **Multiple Rational Agents.** Consider $n > 1$ identical rational investors. First consider the situation in Section 3 in which the uninformed investors do not expect anyone to have superior information and attribute any profit made by the rational investors to pure chance. The aggregate order flow does not impact prices so the rational investors can trade without moving prices against them. The expected profit of the rational investors is limited only by their wealth. It is immaterial whether they trade together or separately. If these rational investors borrow, the contracts they will get are of the form discussed in Section 3, so the expected profit of any rational investor will be constrained by personal wealth even if borrowing is allowed.

Before I consider intermediation, I make a simplifying assumption about the microstructure model. This helps me to obtain monotone comparative statics results which are hard to establish in a general setting. Let $\pi_I(x,\, x^b)$ denote the expected trading profit of the informed investor when his trading strategy is to buy x in case of good private signal (r =1) but the uninformed investors believe that the trading strategy is x^b. In equilibrium, $x = x^b$ and x maximizes $\pi_I(x,\, x^b)$. I assume that the informed investor's expected trading profit $\pi_I(x,\, x^b)$ is decreasing in x^b, the uninformed investors' belief about the informed investor's trading strategy. That is, $\mathrm{d}\pi_I(x, x^b)/\ \mathrm{d}x^b < 0$.

Intuitively, if the uninformed investors think that the informed investor is not trading, that is $x^b = 0$, the order flow has no impact on prices and the informed investor's expected trading profit is large. If the uninformed investors think that the informed investor is trading very aggressively, that is x^b is large, they infer too much information from the order flow and the resulting price reaction reduces the informed investor's expected profit. The assumption says that this relationship is monotonic.

Now, consider the case in which the uninformed investors believe that rational investors have access to a private signal r and that the quality of the signal is q. Suppose each rational investor forms a separate financial intermediary. The question is: how does the number of intermediaries affect their expected profit and average mispricing?

Proposition 10. *With multiple competing financial intermediaries run by rational investors, mispricing persists but decreases in the number of financial intermediaries. The aggregate expected revenue of the intermediaries is decreasing in the number of financial intermediaries.*

The intuition for the result is as follows. When there are multiple financial intermediaries, they compete with each other. This is true regardless of whether the intermediaries' advantage is due to information or rationality. Since the intermediaries operated by rational investors get the same revenue as the intermediaries operated by informed investors, we focus on the latter. Each intermediary trades to maximize trading profit and optimizes the tradeoff between increasing profit by trading more and decreasing profit by revealing information through the order flow. As the number of intermediaries increases, each intermediary thinks that the decrease in profit due to revelation of information through the order flow is shared by all the intermediaries while the increase in profit due to aggressive trading benefits only that intermediary. This results in an increase in the aggregate trading of intermediaries and a decrease in aggregate expected profit when the number of intermediaries increases. This result is similar to the familiar Cournot oligopoly result about increased competition. Since an increase in the number of intermediaries increases aggregate trading of the intermediaries, the order flow becomes more informative and the average mispricing from rational price decreases. Proposition 10 also implies that rational investors will prefer to form a single large intermediary rather than form multiple competing intermediaries. Thus,

Corollary 1. *The rational investors earn greater profit by forming a single financial intermediary than by forming multiple competing financial intermediaries.*

B. **Empirical Implications.** I discuss the empirical implications of the model in this subsection.

***Wealth Requirements for Trading*:** The rational investor in my model forms a financial intermediary in order to relax the wealth constraints that limit his trading. If wealth requirements are small, the rational investor does not need to form a financial intermediary. This predicts that financial intermediation should reduce as the wealth requirements for trading decline. An example of such a change would be the introduction of leveraged securities like options. There are two caveats to this prediction. First, like wealth requirement risk aversion may limit the ability of an individual to take large trading positions. Thus, even if the wealth requirements for trading decline, financial intermediation may allow rational investors to spread risk over a large number of investors. The second caveat is that introduction of leveraged securities increases the number of securities available to liquidity investors for trading. A change in the amount of liquidity trading can impact the profitability of rational investors as discussed below.

***Liquidity Trading*:** An increase in liquidity trading allows the rational investor to hide his trades more effectively and exploit mispricing. Suppose the liquidity trading increases n-fold, so that the new distribution function is $\hat{f}(u) = f(u/n)$. It is easy to show that this allows the informed investor to trade more aggressively. If trading X with wealth W is an ξ_1 equilibrium with the original liquidity trading, then so is trading nX with wealth nW when liquidity trading increases. Since liquidity trading and informed trading increase in the same proportion, the inference problem of the uninformed investors is unchanged. As a result, the level of mispricing remains unchanged but the size of the financial intermediaries, their trading positions and revenue (quantities F, X_I, X_R, and kF in Proposition 4) increase. Thus, my model suggests that if irrational investors introduce predictability in prices, then greater liquidity/noise trading increases the wealth transfer from irrational to rational investors and may decrease the survival chances of irrational investors. Conversely, irrational investors may introduce predictability in prices and yet not lose too much money if the markets are illiquid and prevent profitable informed trading.

***Profitability of Financial Intermediaries*:** While I have not modeled the decision of the rational investor to form a financial intermediary, it will depend on the relevant costs and benefits. The benefit is increasing in the extent of irrationality (Proposition 9) and the extent of liquidity trading. Thus, financial intermediaries will be more profitable when the irrationality among investors is higher and prices more predictable. Further, if short sales constraints are significant, the trading performance of the financial intermediaries relative to individual investors will be negatively correlated with prices or previous market return (Propositions 5, 6).

***Evolution of Market Efficiency*:** The financial anomalies like predictability of return based on factors like book to market ratio, earnings to price ratio or previous returns are often attributed to imperfect rationality of investors. Financial intermediaries exploit the predictability in price (Propositions 4 and 8) and in the process make prices more efficient by reducing predictability. The reduction in predictability is increasing in the size of intermediary's funds (Lemma 4) and in the number of intermediaries (Proposition 10). Thus, the strength of the financial anomalies is negatively correlated with the level of financial intermediation. A new testable empirical prediction is that the financial anomalies should decrease in strength if the size of financial intermediaries increases. A proxy for the strength predictability in prices is the fraction of future return variance that can be explained by the relevant predictive variable (for example, the book to market ratio) and a proxy for the size of financial intermediation is the ratio of the size of financial services sector to the Gross Domestic Product of the economy.

***Contrarian Trading*:** My model predicts that when prices overreact, financial intermediaries will be contrarian investors or will exhibit less momentum trading behavior than individual investors (Propositions 4, 8).

***Welfare Effects*:** My model has no social welfare effects because all investors are risk neutral and there are no surplus-generating or dissipative activities. However, I conjecture that financial intermediation by rational investors will increase social welfare when there is real investment. The intuition is that if the efficiency of investment decisions is related to the informativeness of prices, (for example, as in Boot and Thakor (1997)), a reduction in mispricing is socially desirable and financial intermediaries play an important role in enabling rational investors to reduce mispricing.

6. CONCLUSION

The motivation for this paper is the observation that "behavioral" asset pricing models are characterized by a striking omission, namely that intermediaries are absent in these models. This is at odds with our basic intuition that intermediaries should arise to arbitrage away the deviations in prices from their "efficient-market" values if irrational investors do indeed have (equilibrium) price impact. Thus, the first question I have addressed is: What are the incentives for financial intermediaries in a securities market in which irrational agents have price impact? The insight generated by addressing this question is the following. If the market is dominated by irrational agents in terms of aggregate invested wealth and there is even a single rational agent who is wealth-constrained, a financial intermediary run by the rational agent can arise to profit from the predictability in prices generated by irrational trading, *even if the rational agent's information is no better than that of the irrational agents.* The key is that rationality, which generates higher on-average trading profits, may be confused with superior information (or ability) by irrational agents who refuse to acknowledge that their own irrationality may be the source of their inferior trading profits.

My second question is: can the price impact of irrational agents survive in equilibrium in the presence of the financial intermediary? The answer to this question is yes. The reason is that the intermediary which arises endogenously to exploit mispricing, does *not* wish to eliminate the mispricing that is the source of its trading profit. That is, it imposes on itself a constraint "not to overfish the pond," limiting the size of its investment fund to maximize expected profit. The argument is sustained even with free entry into intermediation, as long as the aggregate wealth of rational agents remains small relative to that of irrational agents.

My model yields several empirical implications. The first implication is that financial intermediaries are more likely to arise and are larger when there are high wealth requirements for trading, for example when option markets are less developed or margin requirements are high. The second implication is that predictability in prices is more likely to be sustained in illiquid markets, that is, markets where adverse price impact of a trade is large. The third implication is that financial intermediation will be more profitable when the predictability of stock prices is higher. Finally, the model predicts that the strength of financial anomalies will decrease as the financial intermediation sector grows.

The notion of combining behavioral finance, asset pricing and financial intermediation is both natural and exciting. This paper represents a modest first step in that it shows that the intuitive idea that introducing rational financial intermediaries could eliminate the potential price impact of irrational agents is not quite right. Thus, the persistence of the effect of irrational trading on equilibrium security prices appears to be robust to the introduction of *endogenously-arising* financial intermediaries. Moreover, intermediation is not predicated on any information advantage for intermediaries. It can arise simply because there is a small island of rationality even in a sea of irrationality.

APPENDIX

Proof of Lemma 1: Consider a game in which the uninformed investors maximize – $E[|\pi_U|]$ where π_U is as defined in (5). A Nash equilibrium for this game will also satisfy Definition 1. The strategy space S_I of the informed investor is the space of functions from {0,1} into [0, W_I] and the strategy space S_U of the uninformed investors is the space of functions from $\Re$ into [0,1]. We endow these spaces with the uniform metric (distance between two functions ξ_1 and ξ_2 is $\sup|\xi_1 - \xi_2|$) and the product space with the product metric. Then, the strategy spaces are compact subsets of a metric space (see Folland (1984), p. 13-16, 115). The payoff functions are continuous (see (4) and (5)) so using Theorem 1.3 of Fudenberg and Tirole (1991), there exists a Nash Equilibrium in mixed strategy. In this mixed equilibrium, the uninformed investors maximize payoff to zero so $E[|\pi_U|] = 0$ for optimal strategies. This is possible only if the strategies being mixed are equal with probability one. Thus, there exists an equilibrium in which the uninformed investors choose a pure strategy. □

Proof of Lemma 2: Substituting equilibrium condition, $\pi_U = 0$ in (5), Since the probabilities lie between 0 and 1, $P(y)$ is expected value of $V^o(s,r)$ and lies between $V^o(s,0)$ and $V^o(s,1)$. When $r = 0$, $E[V^o(s,0)/P(y) - 1] < 0$ so the informed investor maximizes π_I in (4) by choosing $X(0) = 0$. When $r = 1$, $E[V^o(s,1)/P(y) - 1] > 0$ so the informed investor maximizes π_I by choosing $X(1) > 0$. Thus, $E[\pi_I] > 0$. To show that $P(y)$ is an increasing function, it suffices to show that $\Pr(r = 1|y)$ is increasing in y because $V^o(s,1) > V^o(s,0)$.

$$\Pr\left(r = 1\left|y\right.\right) = \frac{f\left(y - X\left(1\right)\right)}{f\left(y - X\left(0\right)\right) + f\left(y - X\left(1\right)\right)} \tag{10}$$

is increasing in $f(y - X(1))/f(y - X(0))$ which is increasing in y because

$$\frac{d}{dy}\frac{f\left(y - X\left(1\right)\right)}{f\left(y - X\left(0\right)\right)} = \frac{f\left(y - X\left(1\right)\right)}{f\left(y - X\left(0\right)\right)}\left(\frac{f'\left(y - X\left(1\right)\right)}{f\left(y - X\left(1\right)\right)} - \frac{f'\left(y - X\left(0\right)\right)}{f\left(y - X\left(0\right)\right)}\right) \geq 0.$$

The last inequality follows because log-concavity implies f'/f is a decreasing function.

□

Proof of Proposition 1: There is no private information so order flow is uninformative. The uninformed investors set price equal to the expected value of δ, which equals the probability that $\delta = 1$. When $s = 1$, the price $V^o(1) = \theta^o > 0.5$ is high and is higher than the rational price $V(1) = \theta$. When $s = 0$, the price $V^o(0) = 1 - \theta^o < 0.5$ is low and is less than the rational price $V(0) = 1 - \theta$. The expected return is $V(1)/V^o(1) < 1$ when $s = 1$ and $V(0)/V^o(0) > 1$ when $s = 0$. □

Proof of Proposition 2: The uninformed investors compete and set π_U in (5) to zero by setting $P(y) = E[\delta|y]$. However, they believe that the order flow is uninformative so $P(y) = E[\delta] = V^o$. The rational investor maximizes his expected profit given by

$$\pi_R(s) = X(s) \cdot [V(s)/V^o(s) - 1] \ . \tag{11}$$

If $s = 0$, $V^o > V$ (see (1), (2)) and the rational investor chooses $X = 0$. If $s = 1$, $V^o < V$ (see (1), (2)), he chooses $X = W_R$. Using (1), (2), and (11), the rational investor's expected profit unconditional on signal s

$$\pi_R = 0.5 W_R \left\{ \frac{1 - \theta}{1 - \theta^o} - 1 \right\}$$

is constrained by his wealth W_R. □

Proof of Proposition 4: Define strategies X_I and P as the strategies $X_I^*(D)$ and $P^*(D)$ associated with the informed trading equilibrium for wealth D. They clearly satisfy conditions 1 and 2 of Definition 2. Define strategy $D(F, k)$ as $D = F$ if $\pi_I(F) \geq kF$ and 0 otherwise. This strategy satisfies condition 3 of Definition 2. The equilibrium trading strategy associated with maximum π_I^* in condition 1 must be the strategy specified in condition 4 as otherwise a lower F will yield same π_I^*. P in condition 5 is the corresponding equilibrium pricing function as it sets π_U to zero in (5). Thus, outcomes X_I and P satisfy strategies $X_I^*(D)$ and $P^*(D)$. The outcome D in condition 3 satisfies the strategy D defined above. To verify condition 4 of Definition 2, we show that F and k maximize the intermediary's revenue. Condition 3 of Definition 2 says that $kD \leq \pi_I^*(D)$ so revenue has an upper bound $\sup(\pi_I^*(D))$. Conditions 1 and 2 of Proposition 4 show that this upper bound is attained so condition 4 of Definition 2 is also satisfied. □

Proof of Lemma 3: If π_I^* is not maximized for finite wealth, then for any given W_I, there exists an equilibrium in which the informed investor buys more than W_I such that the expected trading profit in this equilibrium exceeds the expected profit with trading strategies constrained by wealth W_I. To disprove this we show that expected profit declines when trading strategy is sufficiently large. Consider an equilibrium with trading strategy $X_I(0) = 0$, $X_I(1) = \lambda$. Substituting (10), (11), and $y = u + X_I$ in (4), the informed investor's expected profit is

$$\begin{aligned} \pi_I &= 0.5\lambda \int \left[\frac{V(s,1)\{f(u+\lambda) + f(u)\}}{V(s,0)\, f(u+\lambda) + V(s,1)\, f(u)} - 1 \right] f(u)\,\mathrm{d}u \\ &= 0.5\lambda \{V(s,1) - V(s,0)\} \int \frac{f(u)\, f(u+\lambda)}{V(s,0)\, f(u+\lambda) + V(s,1)\, f(u)} \mathrm{d}u \qquad (12) \\ &= 0.5\lambda \{V(1) - V(0)\}\, A(\lambda). \end{aligned}$$

The integral $A(\lambda)$ decreases with λ and has following bound:

$$\begin{aligned} A(\lambda) &= \int_{-\infty}^{-\lambda/2} \frac{f(u+\lambda)}{V(s,0)\, f(u+\lambda) + V(s,1)\, f(u)} f(u)\,\mathrm{d}u \\ &\quad + \int_{-\lambda/2}^{\infty} \frac{f(u)}{V(s,0)\, f(u+\lambda) + V(s,1)\, f(u)} f(u+\lambda)\,\mathrm{d}u \\ &\leq \frac{1}{V(s,0)} F(-\lambda/2) + \frac{1}{V(s,1)} \{1 - F(\lambda/2)\}. \qquad (13) \end{aligned}$$

Combining (12) and (13),

$$\pi_I \leq 0.5 \{V(s,1) - V(s,0)\} \left[\frac{\lambda \Pr[u < -\lambda/2]}{V(s,0)} + \frac{\lambda \Pr[u > \lambda/2]}{V(s,1)} \right].$$

Since u is integrable, the term in square brackets can be made arbitrarily small by choosing sufficiently large λ. Thus, the informed investor's expected profit becomes arbitrarily small for sufficiently large trading strategies. □

Proof of Proposition 5: First consider $s = 0$. Proposition 4 shows that the intermediary trades F. If liquidity investors demand u, the uninformed investors clear market by selling shares for $y = u + F$ dollars. Substituting $V^o(0,0) = 1 - \theta^o - q$ and $V^o(0,1) = 1 - \theta^o + q$ from (3) in the pricing function in Proposition 4,

$$P(y) = \frac{(1 - \theta^o + q)\, f(y - F) + (1 - \theta^o - q)\, f(y)}{f(y - F) + f(y)}.$$

For each dollar of shares that the uninformed investors sell, they get one dollar and give away $1/P$ shares worth $V(0) = 1 - \theta$ each. Thus, their expected trading profit is

$$\begin{aligned}
\pi_U =& \int (u+F)\left[1 - \frac{(1-\theta)\{f(u) + f(u+F)\}}{(1-\theta^o+q) f(u) + (1-\theta^o-q) f(u+F)}\right] f(u)\,\mathrm{d}u \\
=& F\int \left[1 - \frac{(1-\theta)\{f(u) + f(u+F)\}}{(1-\theta^o+q) f(u) + (1-\theta^o-q) f(u+F)}\right] f(u)\,\mathrm{d}u \\
&+ \int u\left[1 - \frac{(1-\theta)\{f(u) + f(u+F)\}}{(1-\theta^o+q) f(u) + (1-\theta^o-q) f(u+F)}\right] f(u)\,\mathrm{d}u \\
<& \int u\left[1 - \frac{(1-\theta)\{f(u) + f(u+F)\}}{(1-\theta^o+q) f(u) + (1-\theta^o-q) f(u+F)}\right] f(u)\,\mathrm{d}u \le 0.
\end{aligned}$$

The first inequality follows because the first integrand is always negative ($0 \le q \le \theta^o - \theta$). The last inequality follows because u is zero-mean and the expression inside square brackets is a decreasing function of u due to log-concavity of f.

Next consider $s = 1$. Proposition 4 shows that the intermediary does not trade. However, the pricing function in Proposition 4 is increasing in order flow. Thus, liquidity investors lose money to the uninformed investors because price moves against them. Formally, to obtain the uninformed investors' expected profit, we substitute $V^o(1,0)$ and $V^o(1,1)$ from (3) in the pricing function in Proposition 4 and determine the difference in the sale proceeds of the uninformed investors and the value of the shares sold.

$$\pi_U = \int u\left[1 - \frac{\theta\{f(u-F) + f(u)\}}{(\theta^o+q) f(u-F) + (\theta^o-q) f(u)}\right] f(u)\,du \ge 0.$$

The inequality follows because u is zero-mean and the expression inside square brackets is an increasing function of u due to log-concavity of f. □

Proof of Proposition 6: From Proposition 4, when $s = 0$, the intermediary buys the same quantity F that the uninformed investors expect it to buy. Thus, the expected profit per trade equals what the uninformed investors expect. Since the uninformed investors expect the intermediary to trade with probability 0.5 ($r = 1$), the true expected trading profit is twice what the uninformed investors expect; $\pi_R = 2\pi_I$. Investors in the intermediary pay a fee of π_I but earn greater expected trading profit. Also, the intermediary prefers a performance contingent contract to get a share of superior trading profit. When $s = 1$, Proposition 4 shows that the intermediary does not trade. The expected trading profit $\pi_R = 0 < \pi_I$. Thus, the investors in the intermediary pay a fee of π_I but get zero trading profit. Clearly, the intermediary prefers a fixed fee in this case. □

Proof of Proposition 7: Define strategies X_{It} and P_t as the strategies $X^*_{It}(D_t)$ and $P^*_t(D_t)$ associated with the trading equilibrium for wealth D_t. They clearly satisfy conditions 1 and 2 of Definition 3. Define strategy $D_t(F_t, k_t)$ as $D_t = F_t$ if $(1-\alpha)\pi_{It}(F_t) \geq k_t F_t$ and 0 otherwise. This strategy satisfies condition 3 of Definition 3. The equilibrium trading strategy associated with maximum π^*_{It} in condition 1 must be the strategy specified in condition 4 as otherwise a lower F_t will yield same π^*_{It}. P_t in condition 5 is the corresponding equilibrium pricing function as it sets π_{Ut} to zero in (8). Thus, outcomes X_{It} and P_t satisfy strategies $X^*_{It}(D_t)$ and $P^*_t(D_t)$. The outcome D_t in condition 3 satisfies the strategy D_t defined above. To verify condition 4 of Definition 3, we show that F_t and k_t maximize the intermediary's revenue. The part proportional to trading profit is clearly maximized because equilibrium strategies result in $\pi_{It} = \sup \pi^*_{It}$. The fixed fee $k_t D_t \leq (1-\alpha)\pi^*_{It}(D_t)$ is bounded by $(1-\alpha)\sup(\pi^*_{It})$ but this bound is also attained in the equilibrium outcome. Finally, the belief revision of the uninformed investors is rational. If quality of the intermediary's private signal is q, the probabilities of the following outcomes are:

$$s = 0, b = 1, \delta = 1 : 0.25(1-\theta^o+q), \qquad s = 0, b = 1, \delta = 0 : 0.25(\theta^o - q),$$

$$s = 0, b = 0, \delta = 1 : 0.25(1-\theta^o-q), \qquad s = 0, b = 0, \delta = 0 : 0.25(\theta^o + q),$$

$$s = 1, b = 1, \delta = 1 : 0.25(\theta^o+q), \qquad s = 1, b = 1, \delta = 0 : 0.25(1-\theta^o - q),$$

$$s = 1, b = 0, \delta = 1 : 0.25(\theta^o-q), \qquad s = 1, b = 0, \delta = 0 : 0.25(1-\theta^o + q).$$

Removing scalar factor of 0.25 doesn't affect revision process. The expressions can be summarized as $\psi(q) = (2\theta^o - 1)\mathbf{1}_{s=\delta} + 2q\mathbf{1}_{b=\delta} + 1 - \theta^o - q$. Bayes formula yields

$$g_{t+1}(q) = \frac{\psi(q)\, g_t(q)}{\int \psi(\hat{q})\, g_t(\hat{q})\, d\hat{q}}$$

□

Proof of Proposition 8: The uninformed investors cannot distinguish between equilibrium ξ_3 and ξ_4. The conditions 1, 2, 3, and 6 on the uninformed investors' behavior in

Definition 4 are identical to the conditions 1, 2, 3, and 5 on their behavior in Definition 3. The outcomes 1, 2, 3, 4, 5, and 7 that the uninformed investors see or expect in Proposition 8 are identical to the outcomes 1-6 they see or expect in Proposition 7. Thus, the verification of the equilibrium requirements on the uninformed investors' beliefs and strategies follows from the proof of Proposition 7 with identical strategies X_{It}, P_t, and D_t.

We need to show that conditions 4 and 5 in Definition 4 on the intermediary's strategies are satisfied. First, we verify condition 4 which states that the intermediary's trading strategy X_{Rt} is optimal. Clearly, $X_{Rt} = 0$ when $s = 1$ because shares are overvalued. Formally, we have $V_t^o(1,1) \geq P_t \geq V_t^o(1,0) \geq V(1)$ where the last inequality follows from (1), (6) and $q_t \leq \theta^o - \theta$. When $s = 0$, shares are undervalued as $V_t^o(0,0) \leq P_t \leq V_t^o(0,1) \leq V(0)$ from (1), (6) and $q_t \leq \theta^o - \theta$. To see why it is optimal to buy F_t, consider the trading strategy that the uninformed investors expect the intermediary to follow. They expect the intermediary to maximize expected profit by buying F_t when it observes $r = 1$ and has valuation $V_t^o(0,1)$. Since the intermediary's true valuation $V(0)$ is even higher, it must buy at least F_t to maximize profit. Since it cannot buy more than F_t, it buys exactly F_t. With this strategy, $\pi_{Rt} = 0$ if $s = 1$ and $\pi_{Rt} = 2\pi_{It}$ if $s = 0$.

To verify condition 5 of Definition 4, the part proportional to π_{Rt} is maximized because it is proportional to π_{It} (see previous para), which is maximized to $\sup \pi_{It}^*$. The fixed fee $k_t D_t \leq (1-\alpha)\pi_{It}^*(D_t)$ is bounded by $(1-\alpha)\sup(\pi_{It}^*)$ but this bound is also attained in the equilibrium outcome. □

Proof of Proposition 9: The outcomes that the uninformed investors observe in Proposition 8, their probabilities and the likelihood that the uninformed investors assign to them are:

$\Pr(s=0, b=1, \delta=0) = 0.5\theta.$	Likelihood assigned $\propto \theta^o - q.$
$\Pr(s=0, b=1, \delta=1) = 0.5(1-\theta).$	Likelihood assigned $\propto 1 - \theta^o + q.$
$\Pr(s=1, b=0, \delta=0) = 0.5(1-\theta).$	Likelihood assigned $\propto 1 - \theta^o + q.$
$\Pr(s=1, b=0, \delta=1) = 0.5\theta.$	Likelihood assigned $\propto \theta^o - q.$

Thus, they observe with probability θ events to which they assign likelihood $\theta^o - q$ and with probability $1-\theta$, they observe events to which they assign likelihood $1-\theta^o+q$. Clearly, the probabilities match likelihood if $q = q^* \equiv \theta^o - \theta$. The maximum likelihood estimate of q converges to q^* with probability 1 (see Degroot (1970), p. 209). If the prior distribution g_0 is smooth in a neighborhood of q^*, the posterior distribution g_t converges to an approximately normal distribution around q^* (see Degroot (1970), p. 215). The approximation is exact if uninformed investors start with a prior which is flat at q^*, that is $\mathrm{d}g_0(q)/\mathrm{d}q = 0$ at $q = q^*$. □

Proof of Lemma 4: Consider the equilibrium outcome in Proposition 8 for the case $s = 1$. The intermediary does not trade so the order flow y observed by the uninformed investors is u. Substituting this in the equilibrium pricing function, expected price is

$$E[P_t] = \int \frac{(\theta^o + q_t) f(u - F_t) + (\theta^o - q_t) f(u)}{f(u - F_t) + f(u)} f(u)\,\mathrm{d}u$$

Differentiating the expected price with respect to the intermediary's trading position F_t,

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}F_t} E[P_t] &= -2q_t \int \frac{f'(u - F_t) f^2(u)}{\{f(u - F_t) + f(u)\}^2}\mathrm{d}u \\
&= -2q_t \int_{-\infty}^{u^*} \frac{f'(u) f^2(u + F_t)}{\{f(u) + f(u + F_t)\}^2}\mathrm{d}u - 2q_t \int_{u^*}^{\infty} \frac{f'(u) f^2(u + F_t)}{\{f(u) + f(u + F_t)\}^2}\mathrm{d}u \\
&\leq -2q_t \frac{f^2(u^* + F_t)}{\{f(u^*) + f(u^* + F_t)\}^2} \left\{ \int_{-\infty}^{u^*} f'(u)\,\mathrm{d}u + \int_{u^*}^{\infty} f'(u)\,\mathrm{d}u \right\} \\
&= -2q_t \frac{f^2(u^* + F_t)}{\{f(u^*) + f(u^* + F_t)\}^2} \int_{-\infty}^{\infty} f'(u)\,\mathrm{d}u = 0.
\end{aligned}$$

The second equality is obtained by a change of variable and u^* is the point at which f peaks (f is log-concave so it is also unimodal). The inequality follows because $f(u + F_t)/f(u)$ is decreasing in f if f is log-concave. Thus, increasing the size of funds raised decreases average price and reduces overpricing. The proof for the case $s = 0$ is similar.

□

Proof of Proposition 10: Consider $n_2 > n_1 \geq 1$. Let x^* be equilibrium trading strategy of each of the n_2 competing intermediaries. The uninformed investors believe that each intermediary is operated by an investor with private signal. The notation $\pi_I(x, x^b)$ now refers to the aggregate expected profit of all intermediaries when they trade x in aggregate while the uninformed investors believe they are trading x^b. If an intermediary trades x instead of equilibrium strategy x^*, its share of the aggregate expected trading profit is

$$\frac{x}{x + (n_2 - 1)\, x^*} \pi_I \left(x + (n_2 - 1)\, x^*, n_2 x^*\right). \tag{14}$$

For x^* to be an equilibrium trading strategy, incentive compatibility requires that this expected profit be non-decreasing in x at $x = x^*$ (as otherwise each intermediary would prefer to trade less). Differentiation of (14) yields

$$\left(1 - \frac{1}{n_2}\right) \pi_I \left(n_2 x^*, n_2 x^*\right) + \pi_{I1} \left(n_2 x^*, n_2 x^*\right) \geq 0. \tag{15}$$

Here, the notation π_{I1} denotes the derivative of π_I with respect to its first argument. Now consider n_1 competing intermediaries. The condition for $(n_2/n_1)x^*$ to be an equilibrium trading strategy for each of these intermediaries is

$$\left(1 - \frac{1}{n_1}\right) \pi_I \left(n_2 x^*, n_2 x^*\right) + \pi_{I1} \left(n_2 x^*, n_2 x^*\right) \geq 0. \tag{16}$$

If this condition is satisfied, this strategy can be sustained as equilibrium and yields the same aggregate profit as that of n_2 competing intermediaries as the aggregate demand of intermediaries is the same in both cases. However, if (15) holds as an equality (that is n_2 competing intermediaries are not wealth constrained in equilibrium) then (16) is violated. In this case, we shall show that existence of an equilibrium trading strategy $x^0 < (n_2/n_1)x^*$ which yields greater aggregate profit.

Consider function $(1 - 1/n_1)\pi_I(x, x) + \pi_{I1}(x, x)$. The function is negative at $n_2 x^*$ by assumption and positive at 0 because $\pi_{I1}(0, 0) > 0$ (when no investor expects any informed trading, informed trading is profitable). So we can define x^0 to be the largest value less than $n_2 x^*$ at which the function equals zero. This trading strategy is an equilibrium as it satisfies the incentive compatibility condition. Further, the aggregate profit $\pi_I(x^0, x^0) > \pi_I(n_2 x^*, n_2 x^*)$. To see this note that for all x between x^0 and nx^*, $\mathrm{d}\pi_I(x, x)/\mathrm{d}x = \pi_{I1}(x, x) + \pi_{I2}(x, x) < \pi_{I1}(x, x) < (1 - 1/n_1)\pi_I(x, x) + \pi_{I1}(x, x) < 0$.

Thus, the aggregate expected trading profit is non-decreasing in the number of intermediaries if the intermediaries are actually operated by investors with private information. However, the same holds when the intermediaries are operated by rational investors because the uninformed investors cannot distinguish between the two and pay the same revenue. Further, the aggregate order flow is non-decreasing in the number of intermediaries ($n_2 x^*$ for n_2 intermediaries and x^0 for n_1 intermediaries) so using Lemma 4, average mispricing is non-increasing in the number of intermediaries. $\square$

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